# THE POPULATION OF CATACLYSMIC VARIABLE SYSTEMS

Ulrich Kolb
*Department of Physics and Astronomy, The Open University, Walton Hall, Milton Keynes, MK6 7AA, UK*
U.C.Kolb@open.ac.uk

**Abstract** I discuss studies involving cataclysmic variable (CV) population models. I summarize the main elements of CV formation and emphasize the principal difficulties of testing population synthesis calculations against observations. The continuity equation approach to describe the CV orbital period distribution serves as an illustration. I present an update on work studying the $^{26}$Al production by the Galactic nova population, the age of the CV population, an alternative period gap model, and the effect of nova outbursts on the mass transfer rate spectrum. A discussion of the impact on CV research of the new generation of models of low–mass stars and brown dwarfs concludes this review.

**Keywords:** cataclysmic variables, novae

## 1. INTRODUCTION

In cataclysmic variables (CVs) a white dwarf accretes hydrogen–rich matter from a low–mass Roche–lobe filling companion star. Orbital periods have been measured for about 400 systems (Ritter & Kolb 1998; updated). Most periods fall in the observationally "convenient" range of 1-10 hrs, making CVs ideal laboratories for the study of accretion physics (see e.g. Marsh 2000). The fairly well–defined orbital period distribution has prompted numerous statistical studies, aimed at deducing the evolutionary history of CVs from the shape of this distribution. A major obstacle is the need to disentangle genuine evolutionary effects from selection effects. To first order CVs represent a magnitude–limited sample, and most systems reside within 100-200 pc from the Sun. Yet it is often the time–variable flux accompanying accretion phenomena which draws attention to CVs even at much larger distances. Although this limits the use of population synthesis techniques I shall show examples where their application is meaningful and helpful.

## 2. PREHISTORY - A SUMMARY

I start by summarising determining factors in the evolutionary history of CVs.

The white dwarf (WD) primary component is the remnant of the degenerate core region of a giant star which has dimensions much larger than the typical orbital separations of CVs. The progenitor binary must have been much wider and hence must have experienced substantial orbital decay by a factor $10^2 - 10^3$. This is usually attributed to the formation of a common envelope (see Taam & Sandquist 2000, Iben & Livio 1993, for reviews) as a result of runaway mass transfer when the giant progenitor overflows its Roche lobe. The core and the low–mass secondary star effectively orbit inside this common envelope (CE). The envelope gains energy and angular momentum through dynamical friction, and is eventually expelled from the system. It is usually assumed that a fraction $\alpha_{\rm CE}$ (of order unity) of the released orbital energy is used to unbind the envelope,

$$\frac{G(M_c + M_{\rm env})M_{\rm env}}{\lambda R} = \alpha_{\rm CE}\left(\frac{GM_cM_2}{2a_f} - \frac{G(M_c + M_{\rm env})M_2}{2a_i}\right). \quad (1)$$

Here $a_i$ is the initial orbital separation, $M_c$ and $M_{\rm env}$ the core and envelope mass of the giant, $R$ the Roche–lobe radius of the giant at the time of first contact, and $M_2$ the mass of the secondary. The parameter $\lambda$ takes account of the detailed structure of the giant and relates the phenomenological expression on the left hand side of (1) to the actual binding energy. For WD–producing intermediate–mass giants $\lambda$ typically varies between 0.2 and 0.8 (Dewi & Tauris 2000a), while for higher–mass stars $\lambda$ is generally smaller (Dewi & Tauris 2000b). If (1) cannot be satisfied for a positive $a_f$ that would accommodate the secondary inside its own Roche lobe the system is assumed to merge. Although the product $\alpha_{CE}\lambda$ is the only free parameter in the formulation (1), there are other, hidden uncertainties. Without knowledge of the detailed physics of the ejection process it remains unclear to what extent the internal energy contributes to unbind the envelope.

The WD/red dwarf pair emerging from the CE is close but still detached. Subsequent evolution into contact is due to the loss of orbital angular momentum, either by the emission of gravitational waves, or by some other mechanism such as magnetic stellar wind braking (e.g. Verbunt & Zwaan 1981). Commonly used braking models represent an extrapolation of some form of the Skumanich law (Skumanich 1972), the empirical relation describing the rotational braking of single stars. The increasingly complex picture that has emerged through more recent observations (see e.g. Krishnamurthi et al. 1997; Sills et al. 2000; and references therein) still awaits assessment in the context of CVs.

A number of authors have used population models with standard assumptions about the distribution of newly forming binaries in orbital period and component mass space to study the formation of CVs (e.g. de Kool 1992; Han et al. 1995; Politano 1996; Hurley et al. 2000, Howell et al. 2000). These models show that with a standard Skumanich–type magnetic braking law only about 1% of ZAMS binaries with primary mass in the range $1 \lesssim M_p/M_\odot \lesssim 9$ and orbital separation $\lesssim 10^6 R_\odot$ form a CV within a Hubble time.

Once a CV forms successfully it belongs to one of three distinct groups, depending on the nature of the white dwarf. CVs with a low–mass helium WD (with mass $\lesssim 0.45 M_\odot$) descend from binaries where the former giant star filled its Roche–lobe before the ignition of core helium burning. Helium WDs form only through a binary evolutionary channel. The progenitors of CVs with an intermediate–mass or high–mass C/O WD ($\gtrsim 0.55 M_\odot$) have been wide enough to allow the giant to evolve to the AGB stage before it filled its Roche lobe. The mass gap between helium and C/O WDs is due to the core mass growth during core helium burning. Progenitor binaries with primary stars evolving through core C/O burning but avoiding a supernova result in CVs with massive O/Ne/Mg WDs. The minimum mass of O/Ne/Mg WDs is uncertain but probably close to $1.2 M_\odot$ (e.g. Nomoto 1984).

The shape of the initial secondary mass spectrum is dominated by the effect of mass transfer stability (e.g. Ritter 1996), which requires $M_2/M_1 < q_{\rm crit} \simeq 1$ and therefore removes systems with high–mass donor stars. Although it is generally accepted that systems that are unstable against dynamical–timescale mass transfer (unstable systems with secondary mass $\lesssim 0.7 M_\odot$) disappear from the population, i.e. merge, the fate of systems undergoing thermal–timescale mass transfer (initial mass $\gtrsim 0.7 M_\odot$) is less clear (see below). Most population models tend to generate a spike in the formation rate distribution at $M_2 \simeq 0.4 M_\odot$, which is the upper limit for dynamical stability of mass transfer onto a $0.6 M_\odot$ WD, the dominant mass in the population. If thermal–timescale mass transfer systems are included and assumed to "appear" as CVs once their mass ratio is smaller than unity, they produce a large spike of systems forming between 6 and 10 hrs orbital period. Except for these features the initial secondary mass spectrum is roughly flat in $\log M_2$.

As a rule one finds that in order to successfully form a CV the initial mass ratio of the progenitor binary has to be in excess of $3 - 4$ (Politano 1996), to ensure the formation of a common envelope. The initial separations are between $10^2 - 10^3 R_\odot$ (i.e. orbital periods of months to years), so as to establish first contact only after the formation of a helium or carbon/oxygen core. A moderate variation of the parameter $\alpha_{CE}\lambda$, perhaps in the range $0.1 - 1$, simply shifts the parameter space of CV forming binaries from larger to smaller giant radii at first contact, without significantly altering the expected initial WD mass and secondary mass distribution (see e.g. de Kool 1992). The predicted total number of CVs in the Galaxy — assuming that CVs once born never die — is between $10^6$ and a few times $10^7$, translating into local space densities between $10^{-5} - 10^{-4}$ pc$^{-3}$. The space density is smaller if the initial mass ratio distribution is more sharply peaked to unity, as in this case fewer systems obey the initial mass ratio limit.

## 3. THE PERIOD HISTOGRAM - CONTINUITY EQUATION

Given a CV formation rate the continuity equation provides a first estimate of the shape of the CV orbital period distribution, if stationarity and an ideal convergence of evolutionary tracks holds. The latter property refers to the observation that evolutionary tracks of CVs forming with different initial secondary masses converge rather quickly to an essentially unique track in e.g. orbital period $P$ – mass transfer rate $\dot{M}$ space. This behaviour is a consequence of the fact that the angular momentum losses driving mass transfer depend on the current system parameters only, and that low–mass main–sequence stars subject to mass loss rapidly reach a new equilibrium state (Stehle et al. 1996).

If this is the case the continuity equation in orbital period space reads

$$\frac{\mathrm{d}}{\mathrm{d}P}\left\{n(P)\,\dot{P}\right\} = \dot{n}(P), \tag{2}$$

where $n(P)$ is the true (intrinsic) number density of CVs in period space, $\dot{P}$ the local speed in period space due to secular evolution, and $\dot{n}(P)$ denotes the CV formation rate at period $P$ (number density of systems forming per unit time).

The dominant observational selection effect is from the brightness of the system, which in turn is essentially given by the accretion luminosity $L_{\rm acc}$. With $L_{\rm acc} \propto \dot{M}$ the observed number density scales to lowest order as

$$\begin{aligned} n_{obs} &\propto n\,\dot{M}^{\alpha} \\ &= n\,\dot{P}^{\alpha}\left(\frac{\dot{M}}{\dot{P}}\right)^{\alpha} \\ &= n\,\dot{P}^{\alpha}\left(\frac{\mathrm{d}M_2}{\mathrm{d}P}\right)^{\alpha}, \end{aligned} \tag{3}$$

where the selection index $\alpha$ is of order unity (see e.g. Kolb 1996). The derivative $\mathrm{d}P/\mathrm{d}M_2$ is related to the mass–radius index $\zeta = \mathrm{d}\ln R/\mathrm{d}\ln M_2$ of the donor via

$$\frac{\mathrm{d}P}{\mathrm{d}M_2} = \frac{1}{2}\,(3\zeta - 1)\,\frac{P}{M_2}. \tag{4}$$

Given the track convergence, $\mathrm{d}P/\mathrm{d}M_2$ is a known function of $P$. Although weakly dependent on the mass transfer timescale $t_M = M_2/\dot{M}$, $\mathrm{d}P/\mathrm{d}M_2$ mainly reflects features of the mass radius relation of low–mass stars (cf. Kolb 1993).

Using (3) and integrating (2) gives

$$n_{\mathrm{obs}}(P) \propto \dot{P}^{\alpha-1}\left(\frac{\mathrm{d}P}{\mathrm{d}M_2}\right)^{\alpha}(P)\quad\int_{\infty}^{P}\dot{n}(P')\mathrm{d}P' \tag{5}$$

while for $\alpha = 1$ this reduces to

$$n_{\mathrm{obs}}(P) \propto \left(\frac{\mathrm{d}P}{\mathrm{d}M_2}\right)(P)\quad\int_{\infty}^{P}\dot{n}(P')\mathrm{d}P' \tag{6}$$

The last equation shows that if the selection factor is close to unity the shape of the period distribution is governed by two factors, the birth rate integral $\int_{\infty}^{P}\dot{n}(P')\mathrm{d}P'$ and the stellar response term $\mathrm{d}P/\mathrm{d}M_2$. As the formation rate $\dot{n}(P)$ appears only in integrated from it is not surprising that the observed period distribution is insensitive to detailed features of the birth rate distribution. Rather, via the response term the observed distribution might hold clues on stellar structure (Kolb 1993; but see Sect. 4.3 below). If $\alpha$ is significantly different from unity, an additional term involving $\dot{P}$ appears. This term depends on the uncertain angular momentum loss timescale. The main task in interpreting the observed period distribution is therefore to disentangle features due to the birth rate from effects due to the orbital evolution timescale.

As long as observational selection cannot be modelled with greater confidence, the diagnostic value of the observed orbital period distribution, and of population synthesis models trying to reproduce it, is limited.

## 4. APPLICATIONS

Despite this rather pessimistic note there are a wealth of applications where CV population synthesis models are vitally important tools. I give four examples.

### 4.1. THE AGE OF CVS

From the above discussion of the track convergence it is clear that the age of a CV does not affect its mass transfer rate at a given orbital period $P$. Hence the age distribution of CVs is not affected by brightness–selection. Different ages at the same $P$ could come about due to different initial secondary masses, and due to the time spent evolving from different post–CE orbital separations into contact.

Population synthesis models predict the age variation of CVs as a function of $P$ for a given set of population parameters. As an example, Kolb & Stehle (1996) used de Kool's standard model with $\alpha_{rmCE}\lambda = 0.5$, and the magnetic braking law from Verbunt & Zwaan (1981) with the calibration parameter set to unity. They found that systems above the CV period gap ($P \gtrsim 3$ hr) have an average age of 1 Gyr, with most of them being younger than 1.5 Gyr, while systems below the gap ($P \lesssim 2$ hr) display a wide range of all ages above about 1 Gyr, with a mean of 3-4 Gyr. Observational confirmation of such an age difference would be strong evidence for the suspected disparity of the angular momentum loss timescales in systems with low–mass ($\lesssim 0.3M_\odot$) and high–mass secondaries.

In principle, the WD luminosity (or temperature) and the dispersion of CV space velocities could be used as an age indicator.

Assuming standard core cooling, the above CV ages translate into a WD temperature $\simeq 10 \times 10^3$ K and $\simeq 7 \times 10^3$ K for systems above and below the period gap, respectively (for a $0.8M_\odot$ C/O WD, cf. Chabrier et al. 2000; a He WD with mass $0.4M_\odot$ would have a similar temperature, cf. Hansen 1999). The observed WD temperatures are much higher, typically between 10 000 and 20 000 K below the gap, and 20 000 K up to 50 000 K above the gap (e.g. Gänsicke 1999). This suggests that accretional heating dominates over secular core cooling. The observed trend of cooler WDs in CVs with shorter periods could be a simple consequence of the higher mass accretion rate and hence more effective accretional heating above the period gap. This in itself can be regarded as independent evidence for the corresponding difference in angular momentum loss timescales.

A more promising test of the predicted age difference makes use of the empirical relation between age $t$ and space velocity dispersion $\sigma(t)$ of a coeval group of stars. From field stars in the solar neighbourhood Wielen et al. (1977) derive $\sigma(t) = k_1 + k_2 t^{1/2}$ ($k_1, k_2$ are constants). Using this Kolb & Stehle (1996) obtained an intrinsic dispersion of the line of sight velocity of the centre of mass (the $\gamma$ velocity) $\sigma_\gamma \lesssim 20$ km/s above the period gap, while $\sigma_\gamma \simeq 30 - 35$ km/s below the gap. The difference is mainly due to the time spent evolving from the post–CE orbit into contact. If magnetic braking does not operate in this detached phase, then the above values are $\sigma_\gamma \simeq 27$ km/s above the gap versus $\sigma_\gamma \simeq 32$ km/s below the gap.

Measurement errors introduce an additional spread of $\gamma$–velocities and could entirely wipe out the signal. This is the most likely reason why the data compiled by van Paradijs et al. (1995) from the literature do not show any systematic difference between short–period and long–period CVs. Other reasons why the calculated difference might overestimate the actual difference could be that the magnetic braking law assumed by Kolb & Stehle (1996) is a fairly efficient one for systems with secondary mass above $1 M_\odot$ (cf. the discussion in Kalogera et al. 1998), and that the underlying $\sigma(t)$ relation might be less steep as claimed by Wielen et al. (1977). Recent work on stars in the solar neighbourhood by Fuchs et al. (2000) largely confirms the Wielen et al. relation, while Binney et al. (2000) find a somewhat shallower slope $\sigma(t) \propto t^0.33$ from HIPPARCOS stars. In contrast, Quillen et al. (2000) claim that there is no age–dependent dispersion for stars that formed within the last 10 Gyrs.

Despite all these uncertainties a dedicated observational programme to improve on $\gamma$ velocity measurements (North et al. 2000) has already resulted in improved values for four CVs. So far the data are consistent with a small velocity dispersion ($\lesssim 15$ km/s) above the period gap.

## 4.2. ALUMINIUM-26

During a classical nova outburst CVs eject a large fraction of the material that has previously accreted onto the WD into the interstellar medium. The outburst is a result of explosive hydrogen burning, a thermonuclear runaway (TNR), on the WD surface. Although nova ejecta are rich in heavy elements (see e.g. Starrfield 1999) the total contribution of novae to Galactic nucleosynthesis is probably small. Novae are, however, potentially important sources of e.g. $^7$Li and the radioactive isotopes $^{22}$Ne and $^{26}$Al (see Gehrz et al. 1998 for a review).

CV population models are a natural tool to translate nucleosynthesis yields of individual TNR models into the Galactic contribution of the whole nova population. Adding a description of nova outbursts to population models inevitably introduces additional little constrained parameters due to our incomplete understanding of TNR physics. The main problems are the degree of mixing between WD core material and accreted material, and the mechanism of mass ejection. Estimates of the nova shell mass ejected per outburst are therefore rather uncertain.

Here I discuss the production of $^{26}$Al which is detectable via its radioactive decay line at 1.8 MeV. A single TNR on an O/Ne/Mg WDs produces only a small amount of $^{26}$Al. Nonetheless the accumulated contribution by a large number of sources over $10^6$ yrs, the mean lifetime of $^{26}$Al, generates a potentially detectable smooth background emission at 1.8 MeV centred on the Galactic equator. Early COMPTEL all–sky 1.8 MeV maps revealed an unexpected clumpiness (Diehl et al. 1995) which seemed to limit the contribution in a smooth background to less than $1M_\odot$ $^{26}$Al in the Galaxy. This prompted Kolb & Politano (1997) to reevaluate the $^{26}$Al output from a nova population to see if this tighter upper limit constrains CV population models. Their result is expressed in terms of general parameters that allow a rescaling to match a particular dependence of the $^{26}$Al production in an individual outburst on the mass $M_1$ of the white dwarf, on the mass accretion rate $\dot{M}$, and on the degree of mixing. The scaling functions are expressed relative to a base model (a $M_1 = 1.25M_\odot$ WD with $\dot{M} = 10^{17}$ g/s and 50% mixing) which was assumed to give an $^{26}$Al ejecta mass fraction of $9.5\times10^{-2}$, i.e. $6\times10^{-7}M_\odot$ $^{26}$Al per outburst. The preferred population model produced $0.15M_\odot$ $^{26}$Al in the Galaxy, while limits on mixing and the minimum mass of O/Ne/Mg WDs could be established from the observed upper limit of $1M_\odot$.

A more recent revision of proton capture rates (see the summary in Starrfield et al. 1998) led to TNR models with significantly reduced $^{26}$Al output (Starrfield et al. 1998) compared to earlier calculations (Politano et al. 1995). The $^{26}$Al mass fraction in the base model decreased by a factor of 20; the $\dot{M}$ and $M_1$ dependence has not been calculated. With a reduction of the total $^{26}$Al output by this order the currently observed upper limit is too large to place strong constraints on the more interesting CV population parameters. The most recent maps (Plüschke et al. 2000; see also Knödelseder et al. 1999) largely confirm the early limits on $^{26}$Al in a smooth background. However, TNR models by José et al. (1999) do not reproduce this dramatic decrease of $^{26}$Al prodcution when new reaction rates are used. Moreover, José & Hernanz (1998) find a steeper dependence on WD mass mass ($\beta = \mathrm{d}\ln X(^{26}\mathrm{Al})/\mathrm{d}M_1 \simeq 6$ rather than $\beta = 3$, the value preferred by Kolb & Politano 1997) which would increase the total $^{26}$Al output by a factor of a few. They also find a much steeper dependence on the envelope mixing, while $\dot{M}$ has not been varied in their calculations.

Progress may have to await a convergence of predictions from TNR model calculations.

## 4.3. ALTERNATIVE PERIOD GAP MODELS

The disrupted orbital braking model is still the most consistent and convincing explanation for the CV period gap (Spruit & Ritter 1983; Rappaport et al. 1983), the dearth of CVs with orbital periods between 2 and 3 hrs. The model postulates that due to a sudden reduction of the orbital angular momentum loss rate the systems are detached when they evolve through the period range of the gap (see e.g. King 1988 for a review).

The lack of independent observational evidence for a key ingredient — magnetic stellar wind braking ceases to be effective when the donor star becomes fully convective — continues to provoke alternative gap models. A population synthesis procedure provides the means to test these against observations.

One of the latest alternative gap models is by Clemens et al. (1998) who link the gap to a particular property of the mass–radius relation of low–mass main–sequence stars. They derive this relation empirically from a colour magnitude diagram of nearby stars, using bolometric corrections, a temperature–colour scale, and an empirical mass–magnitude relation. The resulting $\log M - \log R$ relation has an unusually small slope $\zeta \simeq 1/3$ in the vicinity of masses $M = 0.15 M_\odot$ and $M = 0.4 M_\odot$ ($\zeta$ is usually between 0.5 and 1). As a consequence, CVs would pile up at the corresponding periods 3.5 hr and 2.0 hr (cf. eq. (4)), causing an apparent dearth of CVs in between, even if mass transfer is continuous.

Kolb et al. (1998) used the mass radius relation by Clemens et al. as an input for a population synthesis model with a continuous orbital angular momentum loss rate (gravitational radiation). The calculated discovery probabilty of systems as a function of orbital period is flat, with two pronounced but rather narrow spikes at the periods quoted above. The shape is insensitive to the selection parameter $\alpha$, cf. (3). This is in contrast to the observed distribution where the number density of systems is much lower in the gap than outside the gap. Hence even if the lower main sequence were as claimed by Clemens et al. (1998) it would not by itself cause the CV period gap.

## 4.4. NOVA OUTBURSTS AND MASS TRANSFER SPECTRA

A long–standing problem in the theory of the secular evolution of CVs is the apparent contradiction between the track convergenge (cf. Sect. 3) and the apparent scatter of observationally deduced mass transfer rates (e.g. Warner 1995, his Fig. 9.8). The track convergence implies that any variation of $\dot{M}$ at a fixed $P$ should largely reflect the dependence of the angular momentum loss rate $\dot{J}$ on the white dwarf mass $M_1$. Since $\dot{M}/M_2 \propto \dot{J}/J$ (e.g. Ritter 1996), and $\dot{J}$ is independent of $M_1$ for mechanisms like magnetic braking which are rooted in the secondary star, the expected dependence is $\dot{M} \propto J^{-1} \propto \sqrt{M_1 + M_2}/M_1$. This is too weak in view of the observed scatter of at least an order of magnitude.

Attempts to reconcile this with the successful period gap model sought to identify physical mechanisms that superimpose short–term fluctuations on the long–term mean mass transfer rate. The fluctuations should not modify the mean transfer rate, otherwise the coherence of evolutionary tracks is lost and collective features such as the period gap would become blurred.

One mechanism discussed extensively in the literature (see Ritter et al. 2000, and refereces therein) is the mass transfer instability induced by anisotropic, weak irradiation of the secondary star. This cannot, by itself, explain the full range of $\dot{M}$ fluctuations, as the instability disappears when the secondary star is predominantly convective. Recently Spruit & Taam (2000) suggested that a circumbinary disc acts as a sink of orbital angular momentum and increases the transfer rate by an amount that depends on the disc surface density distribution, i.e. on the age of the disc. This random element would however destroy the track coherence.

Another mechanism that has been studied before is the effect of nova outbursts on the transfer rate. All CVs are expected to experience some form of nuclear burning of the accreted hydrogen–rich material on the surface of the WD. In most cases this should be explosive, in the form of a classical nova outburst, and hence accompanied by the ejection of the burning layers, the nova shell. The ejection occurs on a very short timecsale ($\lesssim$ years) and effectively leads to an instantaneous change of the orbital separation. The magnitude and sign of the change is determined by the amount of angular momentum the shell removes from the orbit. As the transfer rate varies exponentially with the difference between the donor's stellar and Roche lobe radius $R_L$ (e.g. Ritter 1988), the consequent instantaneous change of $R_L$ causes a discontinuous change of $\dot{M}$. The transfer rate recovers on a timescale set by the thermal relaxation term induced in the donor star by its mass loss history. Previous studies (e.g. Schenker et al. 1998) concluded that the effect on the width of $\dot{M}$ distributions at fixed $P$ are negligible as the transfer rate was found to stay close to the secular mean for most of the time between outbursts, despite amplitudes in excess of 1 dex.

Revisiting the problem, Kolb et al. (2000b) discovered that the mass transfer rate spectrum at given $P$ changes shape when the system is close to dynamical instability and the accreted mass needed to trigger the outburst is in excess of a critical limit of a few $10^{-4}M_{\odot}$. This critical value is a function of WD mass and depends on the photospheric density scaleheight of the donor star. Population synthesis models show that if trigger masses of that order are realised, then the width of transfer rate spectra reach easily 1 dex above 3 hr orbital period. This is largely due to CVs with WDs of mass $0.55 - 0.8 M_{\odot}$ and secondaries with mass close to the stability limit $M_2 \lesssim 0.7 M_1$. Below the period gap the effect is negligible, whatever the trigger mass, as none of the systems is close enough to instability (Kolb et al. 2000b; Kolb 2000). The effect is maximal if there are no frictional angular momentum losses (see Schenker et al. 1998). The crucial question is whether trigger and hence ejection masses are as large as $10^{-4} M_\odot$. TNR calculations usually give values smaller by a factor of 10, while observational estimates are indeed of this order (see e.g. Starrfield 1999).

## 5. THE IMPACT OF NEW STELLAR MODELS

Recent years have seen significant progress in the interpretation of secondary star properties from a new generation of models of low–mass stars and brown dwarfs (see Chabrier & Baraffe 2000 for a review). The models employ an equation of state dedicated to describe the dominant non–ideal plasma effects, and real atmosphere models (Allard & Hauschildt 1998) as an outer boundary condition. This allows one to assign colours and a spectral type to a given stellar model in a self–consistent way.

### 5.1. SPECTRAL TYPE VS ORBITAL PERIOD/DONOR MASS

It has long been known that the spectral type (SpT) of many CV secondaries is considerably later than the spectral type of a main–seqenence star that would fill its Roche lobe at the same orbital period. The new stellar models allow one to quantify this deviation. Baraffe & Kolb (2000) found that the ZAMS forms an upper envelope for the observed location of CV secondaries in the SpT vs $P$ diagram (henceforth SP diagram). Principal causes for a deviation from the ZAMS are (a) thermal disequilibrium due to mass loss, and (b) evolution off the ZAMS prior to the CV stage. In case (a) the SpT is earlier than for equilibrium models if $P \gtrsim 6$ hr, and later if $P \lesssim 6$ hr. This reflects the fact that predominantly radiative stars contract on rapid mass loss and predominantly convective stars expand, while there is little change in effective temperature. In case (b), i.e. for somewhat nuclearly evolved stars, the deviation is in the opposite direction. For $P \gtrsim 5$ hr the SpT is later, for short periods the SpT much earlier than for corresponding ZAMS stars. In both cases the behaviour can be understood by noting that the SpT provides a rough indicator for the donor mass (Kolb et al. 2000a). Any offset from the ZAMS in the SP diagram is largely due to a shift along the $P$ axis to longer $P$ when the star is larger than a ZAMS star of the same mass, or to shorter $P$ when it is smaller. Uncertainties in the theory of convection, made explicit by the difficulty in constraining the value of the mixing length in the standard mixing length theory of convection, make it impractical at present to use the SpT as a quantitative measure of stellar mass (Kolb et al. 2000a).

A comparison of the location of observed CV secondaries in the SP diagram with evolutionary tracks by Baraffe & Kolb (2000) shows that for $3 \lesssim P/\mathrm{hr} \lesssim 6$ the generally late SpT could arise from the effect of mass loss on unevolved stars, but transfer rates in excess of the canonical $2 \times 10^{-9} M_\odot \, \mathrm{yr}^{-1}$ (e.g. Kolb 1996) are needed if the whole range of SpTs were to be explained. For $P \gtrsim 6$ hr the data seem to imply that a large fraction of CVs have an evolved secondary star. A track proceeding with small transfer rate ($10^{-9} M_\odot \, \mathrm{yr}^{-1}$) that starts from a terminal age main–sequence star provides a lower envelope to the data points, lending credibility to this interpretation. Overall, a mass transfer rate that increases with decreasing period, and which is a function of central hydrogen content (as a measure of the age of the secondary) seems most consistent with the overall distribution of systems in the SP diagram, and with the track coherence required by the period gap.

In all these calculations a systematic error may arise from the way a one–dimensional stellar model with a certain mass and radius is translated into a Roche–lobe filling star. Most studies apply the concept of volume–equivalent radii. Here the radius coordinate $r$ of the one–dimensional model is taken to represent the Roche equipotential that encloses the same volume as a sphere with radius $r$. Recent SPH calculations of three–dimensional polytropic stars (Baraffe et al. 2000) have shown that the volume–equivalent radius of this star when it fills its Roche–lobe is larger by a few percent than the equilibrium radius of the star when it is single. The radius increase is slowly increasing with decreasing mass ratio $q = M_2/M_1$, and depends on the polytropic index. For $n = 1.5$ it is $4 - 6\%$, (consistent with results by Uryu & Eriguchi 1999), while for $n = 3$ early indications suggest an even larger effect.

The $n = 1.5$ case is relevant for systems with fully convective secondaries, i.e. for CVs with periods below the period gap, and is discussed in the next section. The $n = 3$ case approximates predominantly radiative stars and would apply to the long–period regime. In the SP diagram it tends to shift tracks to longer periods, alleviating the problem of an uncomfortably large fraction of CV donors that otherwise appeared to be very close to the termination of core hydrogen burning. More detailed work to assess the impact of this radius increase, and the consequent change of the donor star's thermal time, is in progress.

CVs with nuclearly evolved donor stars probably descend from systems with a secondary that is up to three times more massive than the white dwarf. Such systems evolve through a phase of thermal–timescale mass transfer, probably appearing as supersoft X–ray sources. For the interpretation of the SP diagram it is vital to know the outcome of the thermal–timescale mass transfer phase. Baraffe & Kolb (2000) considered initial donor masses up to $1.5 M_\odot$ and found little dependence of the position of systems in the SP diagram below $P \lesssim 10$ hr on the detailed mass loss history at longer periods. A systematic study involving single stars and a bimodal mass transfer rate (constant high rate switches to a constant low rate) shows that larger initial masses give rise to slightly larger deviations from the ZAMS once they emerge from fast mass transfer, while detailed calculations of the thermal–timescale phase (King et al. 2000; Langer et al. 2000) reveal an even more complex picture. A more systematic study involving a self–consistent treatment of the rapid phase is needed.

## 5.2. PERIOD MINIMUM

Theoretical models have consistently failed to reproduce the observed shape of the CV period distribution at the short–period cut–off at 78 min. Pacyński (1971) was the first to relate the cut–off to the minimum orbital period which systems reach when mass transfer is driven by the emission of gravitational waves. The effect is due to the increasing dominance of electron degeneracy pressure slowing down the secondary's radius contraction.

The problem is twofold (see Barker & Kolb 2000): (a) calculations with one–dimensional stellar models place the period minimum at a value that is too short by at least 10%, and (b) a slow–down and reversal of the orbital period derivative implies a "period spike", an accumulation of systems close to the minimum period. There is no evidence for such a feature in the observed period distribution (e.g. Ritter & Kolb 1998).

Using up–to–date brown dwarf models (e.g. Chabrier & Baraffe 2000, and references therein) Kolb & Baraffe (1999) confirmed the small value of $P_{\rm min} \simeq 67$ min (if the white dwarf has mass $0.6 M_\odot$) found in earlier calculations with other codes. Use of rotational and tidal corrections following Chan & Chau (1979) had a negligible effect on $P_{\rm min}$. Adding a contribution of CVs which form with a brown dwarf donor star rather than evolving from non–degenerate higher–mass stars to become a brown dwarf neither improves on the value of the period minimum nor on the expected shape of the distribution. To remove the spike by selection effects requires a selection index $\alpha \simeq 66$, cf. (3), i.e. effectively a discontinuous change of the detectability. As the distribution of magnetic and non–magnetic CVs is statistically indistinguishable, with both showing the cut–off at the same period, the physical cause for such a selection effect is unlikely to be rooted in the accretion disc physics. There is a non–negligible probability that a period spike is not seen in the sample of magnetic CVs due to small number statistics.

At least three potentially promising avenues for progress arise. Both the missing period spike and the apparently too long $P_{\rm min}$ could be explained if CVs avoid the evolutionary phase with $\dot{P} = 0$ altogether.

This would be the case if the systems are detached when the sign of $\dot{P}$ reverses, e.g. as a consequence of disruption of remnant magnetic braking. This could be linked to the observed sharp drop in magnetic activity of low–mass stars at the boundary between spectral type M and L (e.g. Rutledge et al. 2000, and references therein).

Similarly, the phase $\dot{P} = 0$ could be avoided if CVs are much younger than we think. If the 78 min. edge represents the oldest systems in the population which are still on their way down to the true period minimum they would not reach it for another 0.9 Gyrs. In this case essentially no CV should have a donor with mass much less than $0.1 M_{\odot}$. The best candidate CV which appears to be in conflict with this limit is WZ Sge with a claimed mass $M_2 \simeq 0.02 M_{\odot}$ (e.g. Patterson 1998).

The SPH experiments by Baraffe et al. (2000) mentioned above point towards a $4 - 6\%$ underestimate of the secondary star radius for mass ratios and masses appropriate for minimum period CVs. The consequent period increase is smaller than the naive expectation of $6 - 9\%$ due to knock–on effects on the angular momentum loss timescale and thermal time of the donor. Neglecting the latter leads to a decrease of $\dot{M}$ by a factor 1.3 for these tidally expanded stars, and gives $P_{\rm min} \simeq 72$ min. The change of the thermal time is more difficult to account for. If the radius increase is accompanied by a decrease in luminosity as in the case of anisotropically irradiated stars (Ritter et al. 2000), then the effect of the smaller luminosity dominates over the larger radius and gives an overall shorter thermal time $\propto 1/RL$. At the time of writing it is not clear how big this effect is. A period spike would occur in all cases.

## 6. CONCLUSIONS

After a phase of consolidation lacking major new discoveries, independent progress in the understanding of the structure of low–mass stars and brown dwarfs, and in the study of X–ray binaries, has revived interest in the evolutionary history of CVs. Tantalising discoveries show that we are far from a clear understanding of even the main concepts, and further investigations are certain to reveal more surprises.

## Acknowledgments

I thank Isabelle Baraffe, John Barker, Andrew King, Tom Marsh, Klaus Schenker, Saul Rappaport and Hans Ritter for useful discussions, and Andy Norton for improving the language of the manuscript.

## References

Baraffe I., Kolb U. 2000, MNRAS, 318, 354
Baraffe I., Renvoizé V., Kolb U., Ritter H. 2000, in preparation
Barker J., Kolb U. 2000, this volume
Binney J., Dehnen W., Bertelli G., 2000, MNRAS, 318, 658
Chabrier G., Baraffe I. 2000, ARA&A, 38, in press (astro-ph/0006383)
Chabrier G., Brassard P., Fontaine G., Saumon D. 2000, ApJ, 543, 216
Chan K.L., Chau W.Y. 1979, ApJ, 233, 950
Clemens J.C., Reid I.N., Gizis J.E., O'Brien M.S. 1998, ApJ, 496, 352
De Kool M. 1992, A&A, 261, 188
Dewi J.D.M., Tauris T.M. 2000a, A&A, 360, 1043
Dewi J.D.M., Tauris T.M. 2000b, Bormio Proceedings
Diehl R., Dupraz C., Bennett K. et al. 1995, A&A, 298, 445
Fuchs B., Dettbar C., Jahreiss H., Wielen R., in *Dynamics of Star Clusters and the Milky Way*, S. Deiters, B. Fuchs, A. Just, R. Spurzem (eds.), ASP Conf. Series, in press (astro-ph/0009059)
Gänsicke B. 1999, in *11th European Workshop on White Dwarfs*, J.-E. Solheim, E.G. Meistas (eds.), ASP Conf. Series, Vol. 169, 315
Gehrz R.D., Truran J.W., Williams R.E., Starrfield S. 1998, PASP, 110, 3
Han Z., Podsiadlowski P., Eggleton P.P. 1995, MNRAS, 272, 800
Hansen B.M.S. 1999, ApJ, 520, 680
Howell S.B., Nelson L.A., Rappaport S. 2000, ApJ, submitted
Hurley J.R., Tout C.A., Pols O.R. 2000, MNRAS, submitted
Iben I., Livio M. 1993, PASP, 105, 1373
José J., Hernanz M. 1998, ApJ, 494, 680
José J., Coq A., Hernanz M. 1999, ApJ, 520, 347
Kalogera V., Kolb U., King A.R. 1998, ApJ, 504, 967
King A.R. 1988, QJRAS, 29, 1
King A.R., Schenker K., Kolb U., Davies M.B. 2000, MNRAS, in press (astro-ph/0009062)
Knödelseder J., Dixon D., Bennett K. et al. 1999, A&A, 345, 813
Kolb U. 1993, A&A, 271, 149
Kolb U. 1996, in *Cataclysmic Variables and Related Objects*, A. Evans, J.H. Wood (Eds.), Kluwer Academic Publishers, Dordrecht, IAU Coll. 158, p. 433
Kolb U. 2000, in *Evolution of binary and multiple star systems*, P. Podsiadlowski et al. (eds.), PASP, in press
Kolb U., Stehle R. 1996, MNRAS, 282, 1454
Kolb U., Politano M. 1997, A&A, 319, 909
Kolb U., Baraffe I. 1999, MNRAS, 309, 1034
Kolb U., King A.R., Ritter H. 1998, MNRAS, 298, L29

Kolb U., King A.R., Baraffe I., 2000a, MNRAS, in press (astro-ph/0009458)
Kolb U., Rappport S.A., Schenker K., Howell S.B. 2000b, ApJ, in preparation
Krishnamurthi A., Pinsonneault M.H., Barnes S., Sofia S. 1997, ApJ, 480, 303
Langer N., Deutschmann A., Wellstein S., Höflich P. 2000, A&A, in press (astro-ph/0008444)
Marsh T. 2000, in *Astro-tomography workshop*, H. Boffin, D. Steeghs (eds.), Springer
Nomoto K. 1984, ApJ, 277, 791
North R., Marsh T., et al. 2000, in preparation
Pacyński B. 1971, ARA&A, 9, 183
Patterson J. 1998, PASP, 110, 752
Plueschke S., Diehl R., Schönfelder V., et al. 2000, in Proceedings 4th INTEGRAL Workshop, Alicante, ESA-SP, in press
Politano M. 1996, ApJ, 465, 338
Politano M., Starrfield S., Truran J. W., Weiss A., Sparks W. M. 1995, ApJ, 448, 807
Quillen A.C., Garnett D.R. 2000, ApJ, submitted (astro-ph/0004210)
Rappaport S., Verbunt F., Joss P.C. 1983, ApJ,275, 713
Ritter H. 1988, A&A, 202, 93
Ritter H. 1996, in *Evolutionary Processes in Binary Stars*, R.A.M.J. Wijers, M.B. Davies, C.A. Tout (eds.), Kluwer, Dordrecht, p. 223
Ritter H. 2000, NewAR, 44, 105
Ritter H., Kolb U., 1998, A&AS, 129, 83
Ritter H., Zhang Z., Kolb U. 2000, A&A, 360, 959
Rutledge R.E., Basri G., Martín E.L., Bildsten L. 2000, ApJ, 538, L141
Schenker K., Kolb U., Ritter H. 1998, MNRAS, 297, 633
Sills A., Pinsonneault M.H., Terndrup D.M. 2000, 534, 335
Skumanich A. 1972, ApJ, 171, 565
Spruit H.C., Ritter H. 1993, A&A, 124, 267
Spruit H.C., Taam R.E. 2000, ApJ, in press (astro-ph/0010194)
Starrfield S. 1999, Physics Reports, 311, 371
Starrfield S., Truran J.W., Wiescher M.C., Sparks W.M. 1998, MNRAS, 296, 502
Stehle R., Ritter H., Kolb U. 1996, MNRAS, 279, 581
Taam R. Sandquist E. 2000, ARA&A 38, 111
Uryu K., Eriguchi Y. 1999, MNRAS, 303, 329
Van Paradijs J., Augusteijn T., Stehle R. 1996, A&A, 312, 93
Verbunt F., Zwaan C. 1981, ApJ, 100, L7
Wielen R. 1977, A&A, 60, 263