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The evolution of black–hole mass and angular momentum

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ABSTRACT
We show that neither accretion nor angular momentum extraction are likely to lead to significant changes in the mass M_1 or angular momentum parameter a_* of a black hole in a binary system with realistic parameters. Current values of M_1 and a_* therefore probably reflect those at formation. We show further that sufficiently energetic jet ejection powered by the black hole's rotational energy can stabilize mass transfer in systems with large adverse mass ratios, and even reduce the mass transfer rate to the point where the binary becomes transient.

Key words: black hole physics — binaries: close — X–rays: stars — stars: individual (GRO J1655-40, GRS 1915+105, SS433)

1 INTRODUCTION

In a recent paper, Zhang et al. (1997) use the observed strength of the ultrasoft X–ray component in black–hole binaries to estimate the black hole spin. They argue that known systems show a range of spin rate $a_* = a/M_1$ (see their Table 2), where the Kerr parameter $a = cJ_1/GM_1$ with M_1, J_1 the black–hole mass and angular momentum. In particular the two superluminal jet sources GRO J1655-40 ($a_* \simeq 0.93$) and GRS 1915+105 ($a_* \simeq 0.998$) are claimed to spin at rates close to the maximum value $a_* = 1$, while systems such as the soft X–ray transient GS 2000+251 have $a_* \simeq 0$.

The obvious question is whether this claimed range of spin rates reflects systematic spinup from $a_* = 0$, or spindown from $a_* = 1$, or the accidents of birth. In this paper we shall show that neither spinup nor spindown is likely to account for the range. The total mass accreted over the lifetime of any binary black hole is too small to increase a_* from 0 to 1. By contrast, spindown of a black hole from an initial state with $a_* = 1$ is a relatively efficient process if the rotational energy is used to power moderately relativistic jets. However, significant spindown requires the hole to have released improbably large amounts of energy into the surrounding interstellar medium.

It seems therefore that binary black holes retain rather similar spin rates to those they had at birth. The same appears true of their masses: although a loss of rotational energy from a black hole implies that its gravitating mass decreases, we shall see that with realistic energy extraction efficiencies this effect is rather small. We conclude that binary black holes essentially retain the masses and spin rates they were born with. A further interesting result emerges from our analysis: moderately relativistic jet ejection is able to stabilize mass transfer in binaries where the mass ratio would otherwise lead to unstable Roche lobe overflow. Sufficiently energetic jet ejection can even lower the mass transfer rate to the point where the system becomes transient.

2 TOTAL MASS TRANSFER IN BLACK–HOLE BINARIES

A black hole can change its spin either by accreting mass and the associated angular momentum, or by giving angular momentum to matter in its close vicinity. If the hole is a member of a binary system both of these processes are limited by the total mass transferred in the system's lifetime. In this section we estimate this quantity for all types of black–hole binary. We can divide these into high–mass systems, where mass is accreted from the stellar wind of an early-type companion, and low–mass systems, where a late–type star fills its Roche lobe. We may further subdivide the latter group into those where the mass transfer is driven by the nuclear or thermal expansion of the secondary (n–driven), and those where the driving mechanism is angular momentum loss (j–driven).

2.1 Wind–fed binaries

In this type of binary the black hole generally accretes only a small fraction $f = \dot M_{\rm acc}/\dot M_{\rm w}$ of the total mass lost by the companion, usually a supergiant, in the form of a wind at a rate $\dot M_{\rm w}$. A rough estimate for f is

$$f \simeq \frac{r_a^2}{4a^2} \simeq \frac{GM_2^2}{a^2 v_{\rm w}^4} \simeq 2\times 10^{-3}\left(\frac{m_2}{P_d}\right)^{4/3}\left(\frac{v_{\rm w}}{1000\,{\rm km\,s^{-1}}}\right)^{-4} \quad (1)$$

(e.g. Bhattacharya & van den Heuvel 1991), where $m_2 = M_2/\mathrm{M}_\odot$ is the companion mass in solar masses, a the orbital separation, P_d the orbital period (in days), r_a the accretion radius and G the constant of gravitation. With wind velocities $v_\mathrm{w} \lesssim 1000$ km s^{-1} (e.g. Behnensohn et al. 1997) we find typically $f \lesssim 10^{-2}$.

Even with extreme assumptions it is clear that the total transferred mass M_tr cannot exceed a quantity of order

$$M_\mathrm{tr}(\mathrm{wind}) \sim 0.1\mathrm{M}_\odot \qquad (2)$$

2.2 Roche–lobe overflow systems

In such systems a significant fraction of the secondary mass M_2 can be transferred, i.e.

$$M_\mathrm{tr}(\mathrm{Roche}) \lesssim M_2, \qquad (3)$$

subject only to the requirement that mass transfer should remain stable (i.e. does not become so rapid that the system enters a common–envelope phase). This mass is transferred on a timescale t_ev, where t_ev is the evolution time of the binary in the mass–transferring stage. This is given by the angular momentum loss time t_j for j–driven systems (donor a main-sequence star and $M_2 \lesssim 1.5\mathrm{M}_\odot$), and either the thermal timescale for donors crossing the Hertzsprung gap (Kolb et al., 1997; Kolb, 1998) or the nuclear lifetime t_n if the donor is a low–mass giant or a main–sequence star with mass $\gtrsim 1.5\mathrm{M}_\odot$. Not all of the transferred mass may be accreted by the black hole however, as the accretion rate cannot consistently exceed the Eddington value $\dot{M}_\mathrm{Edd} = 10^{-8} m_1$ M$_\odot$ yr^{-1}, where $m_1 = M/\mathrm{M}_\odot$. An upper limit to the *accreted* mass is therefore in all cases given by

$$M_\mathrm{acc}(\mathrm{Roche}) = \min[M_2,\ 10^{-8} m_1 t_\mathrm{ev} \mathrm{M}_\odot] \qquad (4)$$

In practice this means $M_\mathrm{acc} \lesssim M_\mathrm{tr} < M_2 < 1.5\mathrm{M}_\odot$ for j–driven (short–period) systems. For n–driven systems we plot an estimated upper limit for M_acc as a function of initial donor mass in Fig. 1 (case A mass transfer, donor on the main sequence) and Fig. 2 (case B mass transfer, post–core hydrogen but pre–core helium burning phase). The initial black hole mass is $8\mathrm{M}_\odot$ in all cases; systemic angular momentum losses are assumed to be negligibly small. We used simple fitting formulae to describe the variation of global stellar parameters along single–star tracks as given by Tout et al. (1997). To calculate the case A mass transfer rate the donor's radius expansion rate $K = \mathrm{d}\ln R/\mathrm{d}t$ was approximated by that of a single star with the donor's age and current mass. For case B the core mass growth determines the radius variation with little sensitivity to the total mass (e.g. Webbink et al. 1983 for low–mass stars; Kolb 1998 for stars of higher mass), hence we used $K(t)$ from a single star with the donor's initial mass to estimate the transfer rate. The accreted mass M_acc shown in Figs. 1 and 2 represents an upper limit as mass transfer could begin at a later phase than assumed (donor on the ZAMS for case A, on the terminal main sequence for case B). A system formally terminating case A mass transfer when the donor arrives at the terminal main sequence would continue to transfer mass via case B. The case B phase terminates when core helium burning begins or the donor's envelope is fully lost. Any mass transferred in excess of the Eddington rate was assumed to leave the system with the black hole's specific orbital angular momentum, otherwise mass transfer was taken to be conservative.

3 CHANGING THE BLACK–HOLE SPIN BY ACCRETION

Here we consider the effect of accretion in changing the black hole mass and spin. We assume first that black–hole spinup occurs by accretion from a disc terminating at the last stable circular orbit. The accreting matter adds both its rest–mass and its rotational energy to the hole, increasing both the gravitating mass–energy M_1 of the hole and its angular momentum J_1. Bardeen (1970; see also Thorne, 1974) showed that these quantities increase as

$$\Delta M = 3M_i[\sin^{-1}(M_1/3M_i) - \sin^{-1}(1/3)], \qquad (5)$$

$$a_* = \left(\frac{2}{3}\right)^{1/2} \frac{M_i}{M_1}\left[4 - \left(\frac{18M_i^2}{M_1^2} - 2\right)^{1/2}\right]. \qquad (6)$$

Here ΔM is the rest–mass added to the hole from the initial state (assumed to be $M_1 = M_i, a_* = 0$). Once this is such that $M_1/M_i = 6^{1/2}$ we see from (6) that $a_* = 1$. In practice the value of a_* stops slightly short of this maximal value, and further accretion simply maintains this state (Thorne, 1974). From (5) we see that the required additional rest–mass is

$$\begin{aligned}\Delta M &= 3[\sin^{-1}(2/3)^{1/2} - \sin^{-1}(1/3)]M_i \\ &\simeq 1.85M_i = 0.75M_1\end{aligned} \qquad (7)$$

Figure 3 shows a_* as a function of ΔM (given by numerically combining eqs. 5, 6). In order to spin up from $a_* = 0$ to a value $\simeq 1$ the black hole must have accreted a rest–mass of order 75% of its current gravitating mass. This is therefore a lower limit to the accreted mass M_acc.

The resulting lower limits to M_acc for the two presumed rapidly–spinning systems GRS 1915+105 and GRO J1655-40 can be compared with the upper limits obtained from considerations as in Section 2 (see also Tab. 1):

The binary period and component masses of GRS 1915+105 are not known. Zhang et al. (1997) and Cui et al. (1998) find that consistency between black hole parameters derived from the X–ray spectrum and from the 67 Hz QPO (Morgan, Remillard & Greiner 1997), when interpreted either as a trapped g–mode oscillation or a disc precession from the frame–dragging effect, implies a high–mass near–Kerr black hole ($M_1 \simeq 30\mathrm{M}_\odot$, $a_* \simeq 0.998$). If this is true the lower limit on $\Delta M \gtrsim 22\mathrm{M}_\odot$ required for spinup is much larger than what could have been transferred even in the most favourable evolutionary configuration. Repeating the calculations shown in Figs. 1 and 2 for a $30\mathrm{M}_\odot$ primary gives $M_\mathrm{acc} \lesssim 4\mathrm{M}_\odot$ (case A) and $\lesssim 1.5\mathrm{M}_\odot$ (case B).

Conversely, in GRO J1655-40 the binary parameters are reasonably well determined (Orosz & Bailyn, 1997; van der Hooft et al., 1998; Phillips et al. 1998). To find an upper limit to M_acc we assume that the past evolution was conservative, i.e. that $M_1^3 M_2^3 P =$ const. Then the present system parameters as given by Orosz & Bailyn (1997), $M_1 \simeq 7\mathrm{M}_\odot$, $M_2 \simeq 2.3\mathrm{M}_\odot$, $P = 2.62$ d, imply a minimum period $P \simeq 1.08$ d. This is somewhere in the middle of the main sequence band (see e.g. Fig. 1) where the donor's mass–radius index ζ relevant for stability against thermal–timescale mass transfer is $\zeta \simeq 0$ (Hjellming 1989). In this case mass transfer stability demands that initially $(M_2/M_1)_i < 5/6$, see (22) below, hence initially $M_2 \lesssim 4.2\mathrm{M}_\odot$, and therefore $M_{\rm acc} \lesssim 1.9\mathrm{M}_\odot \simeq 0.27\,M_1$. (If M_2 were larger thermal–timescale mass transfer would ensue, at a rate $\simeq M_2/t_{\rm KH} = f_{\rm Edd}\dot M_{\rm Edd}$ with $f_{\rm Edd} = 10 - 100$; $t_{\rm KH}$ is the donor's Kelvin-Helmholtz time. The black hole would accrete only a very small fraction $\simeq 1/f_{\rm Edd}$ of any transferred mass in this phase). Using Phillips et al.'s (1998) lower limits for the present component masses in GRO J1655-40 ($M_1 = 4.2\mathrm{M}_\odot$, $M_2 = 1.4\mathrm{M}_\odot$) gives a minimum orbital period of 1.11 d and also $M_{\rm acc} \lesssim 1.9\mathrm{M}_\odot \simeq 0.27\,M_1$. This value is clearly inconsistent with the spinup requirement $\Delta M > 0.47\,M_1 = 3.3\mathrm{M}_\odot$ if $a_* \simeq 0.93$, the value preferred by Zhang et al. (1997), and only barely consistent with their lower limit $a_* > 0.7$ which requires $\Delta M \gtrsim 0.27\,M_1 \simeq 1.9\mathrm{M}_\odot$.

We conclude that the claimed range of a_* cannot be achieved by spinup of the black hole from an initially non–rotating state.

Much attention has recently been paid to suggestions that quiescent soft X–ray transient (SXT) systems might have higher accretion rates than previously thought, because a large mass flux might be advected into them, i.e. accreted at low radiation efficiency. Advective flows have lower specific angular momentum than the Kepler value and so are even less effective in spinning up the black hole. Advection therefore does not change the conclusion of the last paragraph concerning spinup. However, since the advected specific angular momentum is so low, one might consider the opposite possibility, i.e. reducing a_* to a value close to zero by diluting the original angular momentum. However, even if the advected matter has zero angular momentum we have $a_* = J_{\rm i}/M_1^2 = a_*({\rm i})(M_{\rm i}/M_1)^2$, where M_i, J_i, $a_*(i)$ specify the hole's initial mass, angular momentum and Kerr parameter, respectively. Reducing a_* from 1 to $\simeq 0.1$ in this way requires the black hole mass to increase by a factor $\simeq 3$. Thus the transferred mass must satisfy $M_{\rm tr} \gtrsim 2M_1/3$, again far too large compared with the limits (2, 4).

4 CHANGING THE BLACK–HOLE SPIN BY EJECTION

A rotating black hole can lose angular momentum and rotational energy because of the existence of an ergosphere outside its event horizon. The energy loss implies that the gravitating mass of the hole must decrease, according to

$$\mathrm{d}M_1 = \epsilon\Omega_{\rm H}\mathrm{d}J \tag{8}$$

For convenience in this Section we use geometrized units, in which $G = c = 1$. Here

$$\Omega_{\rm H} = \frac{a_*}{2M_1[1 + (1 - a_*^2)^{1/2}]} \tag{9}$$

is the apparent angular velocity of the horizon (e.g. Misner et al, 1973), and ϵ measures the efficiency of energy extraction. $\epsilon = 1$ is the maximum value, and corresponds to a 'reversible' transformation, in which the area of the event horizon is held fixed, while efficiencies $\epsilon \lesssim 0.5$ are typical for astrophysically realistic processes (e.g. Blandford & Znajek, 1977). Since

$$\mathrm{d}J = \mathrm{d}(a_* M_1^2) = M_1^2\mathrm{d}a_* + 2a_* M_1\mathrm{d}M_1 \tag{10}$$

we get with (8, 9)

$$\frac{2\mathrm{d}M_1}{M_1} = \frac{\epsilon a_*\mathrm{d}a_*}{1 - \epsilon a_*^2 + (1 - a_*^2)^{1/2}}. \tag{11}$$

This can be integrated using the substitution $v = (1-a_*^2)^{1/2}$, and gives finally

$$M_1^2 = M_{\rm max}^2[(1 - a_*^2)^{1/2} + 1]^n\left[\frac{\epsilon(1 - a_*^2)^{1/2} - \epsilon + 1}{1 - \epsilon}\right]^m \tag{12}$$

where

$$n = -\frac{\epsilon}{2\epsilon - 1}, \tag{13}$$

$$m = -\frac{\epsilon - 1}{2\epsilon - 1} \tag{14}$$

and the hole starts with $M_1 = M_{\rm max}, a_* = 1$. The maximum rotational energy extraction is given by setting $a_* = 0$ in (12), which gives

$$M_1^2 = 2^n(1 - \epsilon)^{-m}M_{\rm max}^2. \tag{15}$$

Taking the limit $\epsilon \to 1$ gives $M_1^2 = M_{\rm max}^2/2$, i.e. M_1 reaches its 'irreducible' value $M_{\rm max}/\sqrt{2}$ (Christodoulou 1970, Christodoulou & Ruffini 1971). This maximal energy yield implies a reversible transformation in which the event horizon area is held constant. However in general the energy yield is considerably smaller than this limit. Fig. 4 shows the extracted fractional energy $\Delta m = (M_{\rm max} - M_1)/M_{\rm max}$ as a function of ϵ. We see that for typical efficiencies $\epsilon \lesssim 0.5$ rather less than about 10% of the initial rotational energy is extracted; the hole's mass thus remains very close to its original value $M_{\rm max}$. Physically what is happening is that all of the hole's *angular momentum* is extracted, but the associated *energy* is used inefficiently: the extraction process allows much of this to disappear down the hole, reducing the loss of gravitating mass.

The remaining energy must appear in some form outside the hole. We note that both the systems of Table 1 with large claimed a_* are ejecting relativistic jets. These remove energy at the rate $\Gamma\dot M_{\rm ej}c^2$, where Γ is the specific energy of the jet matter, and $\dot M_{\rm ej}$ is their mass–loss rate. If all the energy of the jet material is in its bulk motion, Γ is simply the Lorentz factor γ of this motion. However, if the spin energy of the black hole is used in other ways, e.g. to excite relativistic electrons, Γ will exceed γ. Thus spindown wins out over spinup as a way of altering a_* because the effect of transferring rest mass $M_{\rm tr}$ from the companion is enhanced by a factor Γ in the former case. If all the transferred matter is ejected in this way until the hole is spun down, the requirement on the total transferred mass becomes

$$M_{\rm tr} = \frac{\Delta m}{\Gamma}M_1. \tag{16}$$

Taking $\Delta m \sim 0.1$, $M_1 = 10\mathrm{M}_\odot$ and bounding Γ below with the value $\gamma \sim 2.55$ inferred for GRO J1655-40 (Hjellming & Rupen 1995) we get a limit $M_{\rm tr} \lesssim 0.4\mathrm{M}_\odot$ on the mass which must be transferred to reduce the spin to zero. Spindown from $a_* = 1$ to $a_* = 0$ therefore seems possible for systems with Roche–lobe overflow. However, if this has occurred in a given system, one would expect to see abundant evidence of the effects of the total extracted energy $\Delta m M_1 c^2 \simeq 10^{54}$ erg on the surrounding interstellar medium. GRO J1655-40 would deposit this energy over a time of $\simeq 10^6$ yr, see (26) below. The mean energy output rate $\simeq 8\times10^6 L_\odot$ is comparable to the luminosity of a cluster of 10 O/B supergiants. The detectability of such an energy deposition depends on the structure and density of the local interstellar medium. The fact that none of the claimed $a_* \simeq 0$ systems shows such evidence suggests that jets are produced only over a relatively short fraction of the system's lifetime. In this case a_* is likely to remain close to its original value ($\simeq 1$).

5 BINARY EVOLUTION WITH RELATIVISTIC JETS

We have seen above that the production of relativistic jets by a rotating black hole can lead to a large loss of gravitating mass from this object, i.e. $\dot M_1(\mathrm{jet}) = -\Gamma \dot M_{\mathrm{ej}}$, where $\dot M_{\mathrm{ej}}$ is the rest mass the jet carries away per unit time. As the jet ejection is a consequence of mass accretion, the ejected rest–mass must be roughly equal to the transferred rest–mass from the companion. Hence we expect that the ejection parameter η, defined by $-\dot M_{\mathrm{ej}} = \eta \dot M_2 < 0$, is of order unity. (Based on energy considerations for the observed jet ejection, Gliozzi et al. 1998 argue that η is close to 1 in GRS 1915+105). In addition the jets will presumably carry off the specific orbital angular momentum $j_1 = M_2 J/M_1 M$ of the black hole from the binary orbit, where M, J are the total binary mass and orbital angular momentum. The jet ejection process therefore constitutes a 'consequential angular momentum loss' or CAML process. The effects of CAML on Roche–lobe–filling systems were studied quite generally by King & Kolb (1995; henceforth KK95), who specified them in terms of mass loss and angular momentum loss parameters α, ν, with

$$\dot M_1 = (\alpha - 1)\dot M_2, \ \frac{\dot J_{\mathrm{CAML}}}{J} = \nu \frac{\dot M_2}{M_2}. \tag{17}$$

Thus here we have

$$\alpha = \eta\Gamma, \ \nu = \frac{\eta\Gamma M_2^2}{M_1 M}. \tag{18}$$

The CAML process can only amplify (or damp) an already–existing mass transfer process driven by angular momentum losses $\dot J_{sys}$ (j–driven systems; $\dot J_{sys}$ is the 'systemic' rate given by e.g. gravitational radiation or magnetic braking) or a nuclear expansion rate $\dot R_2 > 0$ (for n–driven systems). Defining the evolution time $t_{\mathrm{ev}} = |J/\dot J_{\mathrm{sys}}|$ or $(2R_2/\dot R_2)_{M=\mathrm{const.}}$ and repeating the algebra of KK95 gives

$$\dot M_2 = -\frac{M_2}{D t_{\mathrm{ev}}}, \tag{19}$$

where

$$D = \frac{1}{2}\left[\zeta + 2 + \beta_2 - \frac{2M_2}{M_1} + \frac{M_2}{M} + (\eta\Gamma - 1)\frac{M_2}{M_1}\left(\frac{M_1}{M} - \beta_2\right)\right]. \tag{20}$$

(This is a slight generalization of KK95 in the case of n–driven systems.) Here $\beta_2 \equiv \mathrm{d}\ln f_2/\mathrm{d}\ln(M_2/M_1)$ is the logarithmic derivative of the ratio $f_2 = R_L/a$ (the donor's Roche lobe radius R_L in units of the binary separation) with respect to the mass ratio. If $M_2 \gtrsim M_1$ as in KK95 we have $\beta_2 \simeq -M_1/3M$, hence

$$D = \frac{5}{6} + \frac{\zeta}{2} - \frac{M_2}{M_1} + \eta\Gamma\frac{2M_2}{3M}, \tag{21}$$

which is eq. (16) of KK95 with α, ν form our eq. (18). In general, the donor's mass–radius index $\zeta = \partial \ln R_2/\partial \ln M_2$ appearing in (20,21) depends on the secondary's full internal structure and cannot be expressed in closed form. But in many cases an estimate is available. For slow mass transfer and j–driven or n–driven evolution with a main–sequence donor (case A) we have $\zeta \simeq \zeta_{\mathrm{eq}} \simeq O(1)$ (ζ_{eq} is the mass–radius exponent evaluated under the assumption of thermal equilibrium). For n–driven case B evolution slow mass transfer corresponds to $\zeta \simeq 0$. If mass transfer is rapid ζ approaches ζ_{ad}, the mass–radius index evaluated with constant entropy profile in the star. Fully convective stars or stars with a deep convective envelope have $\zeta_{\mathrm{ad}} = -1/3$. Figure 5 shows D from (20) as a function of mass ratio, for various values of Γ (with $\zeta = 0$, $\eta = 1$). Eggleton's approximation for f_2 (Eggleton 1983) was used to calculate β_2.

Comparing with the case with no CAML (i.e. $\alpha = \nu = 0$; conservative mass transfer)

$$D = D_0 = \frac{5}{6} + \frac{\zeta}{2} - \frac{M_2}{M_1} \tag{22}$$

(for $M_2 \lesssim M_1$) and neglecting any change in ζ and t_{ev} we see that the mass loss from the black hole always reduces the mass transfer rate (by trying to widen the binary). With CAML the Roche lobe expands faster (or contracts slower) than without CAML for a given transfer rate; hence $|\dot M_2|$ adjusts to a smaller value. For M_2 significantly smaller than M_1 the reduction in $-\dot M_2$ is small unless

$$\eta\Gamma \gg \frac{3M}{2M_2}. \tag{23}$$

If this holds we have

$$\dot M_2 \simeq -\frac{3M}{2\eta\Gamma t_{\mathrm{ev}}}. \tag{24}$$

Figure 6 shows the variation of the reduction factor $D(\Gamma)/D_0$ with mass ratio (assuming $\zeta = 0 = \mathrm{const.}$, $\eta = 1$). The reduction is largest close to where $D \to 0$ with conservative mass transfer. This signals instability against dynamical–timescale (if $\zeta = \zeta_{\mathrm{ad}}$) or thermal–timescale mass transfer (if $\zeta = \zeta_{\mathrm{eq}}$), with ensuing transfer rates much in excess of values indicated by (19) with $\zeta = O(1)$. A very high transfer rate may cause a common envelope phase which could destroy the system (but see King & Ritter 1998). However, jets actually stabilize mass transfer in systems where a large mass ratio $q = M_2/M_1$ would make mass transfer unstable ($D_0 < 0$) without the jet losses. Mass transfer is stable if the mass ratio is smaller than q_{crit}, where $D(q_{\mathrm{crit}}) = 0$. From (21) we obtain the positive root of $D = 0$ as

$$q_{\mathrm{crit}} = \frac{1}{2}\left[C + \frac{2\eta\Gamma}{3} - 1 + \sqrt{(C + \frac{2\eta\Gamma}{3} - 1)^2 + 4C}\right], \tag{25}$$

where $C = 5/6 + \zeta/2$. A reasonable approximation for $-1/3 \lesssim \zeta \lesssim 10$ is $q_{\mathrm{crit}} \simeq (\eta\Gamma/2.82)^2 + (5 + 3\zeta)/6$. Systems that are unstable against conservative mass transfer but stabilized by jet–induced CAML will still encounter the instability at the end of the jet phase. The mass loss rate from the hole is much larger than the mass loss rate from the donor, so that the mass ratio M_2/M_1 increases during the jet phase.

The maximum duration Δt of the jet–induced CAML phase depends only weakly on $\eta\Gamma$, unlike the transfer rate. As shown above, the extractable gravitating mass from a

Kerr black hole with gravitating mass M_1 is limited by $\Delta m M_1$, where Δm depends on the efficiency of the extraction process. The corresponding limit $\Delta M_{\rm tr} < \Delta m M_1/(\eta\Gamma)$ for the transferred rest–mass during the jet phase translates into an upper limit $\Delta t < \Delta M_{\rm tr}/(-\dot M_2)$ for the duration of this phase. Using (24) as a representative value for $\dot M$, and 0.1 as a typical value for ΔM, we have

$$\Delta t \lesssim 0.1 \frac{M_1}{M} t_{\rm ev}. \qquad (26)$$

As is well known (van Paradijs 1996; King, Kolb & Burderi 1996; King, Kolb & Szuszkiewicz 1997) a low transfer rate is a necessary condition for the system to appear as a soft X–ray transient. This is both empirically true, and expected from the disc instability picture. Black–hole systems with low M_2 all have $-\dot M_2 \sim M_2/t_{\rm ev}$ small enough to satisfy this, whether j–driven or n–driven (King, Kolb & Szuszkiewicz, 1997), i.e. all low–mass black hole systems are transient, even without the extra effect of the jet losses. Values $\Gamma \gtrsim 10$ could however allow a black hole system with a higher–mass companion nevertheless to appear as a transient.

A possible case in point is GRO J1655-40, whose secondary may have a mass $M_2 \simeq 2{\rm M}_\odot$ (Orosz & Bailyn 1997; van der Hooft et al. 1998; Phillips et al. 1998) and appears to be crossing the Hertzsprung gap. Although the secondary is close to a regime where its radius expansion temporarily slows, producing a low transfer rate and transient behaviour (Kolb et al., 1997, Kolb 1998), current estimates of its parameters consistently show it to be too hot to be in this regime. Instead one expects $t_{\rm ev} \sim t_{\rm KH} \sim 10^7$ yr, where $t_{\rm KH}$ is the thermal timescale of the secondary's main–sequence progenitor, so $-\dot M_2 \sim M_2/t_{\rm ev} \sim$ few $\times 10^{-7}/D$ ${\rm M}_\odot{\rm yr}^{-1}$. Without the CAML effect of the jet losses we would have $D \sim 1$, making $-\dot M_2$ far higher than the estimated critical rate $\dot M_{\rm crit} \simeq 4\times10^{-8}$ ${\rm M}_\odot{\rm yr}^{-1}$ (from King, Kolb & Szuszkiewicz, 1997; eqn. 7). However, if the jets are very energetic ($\Gamma \gtrsim 50$) the transfer rate would be reduced by more than the required factor $\simeq 10$ for the claimed mass ratio $\simeq 0.3$ (Orosz & Bailyn 1997), see Fig. 5. Hence if $\Gamma \gtrsim 50$ the system would appear as a transient. This of course requires the jets to be considerably more relativistic than implied by their bulk motion ($\gamma \simeq 2.55$). The occurrence of transients of this type then depends purely on the physics of the jets, and cannot be predicted with current theory. On the other hand, if the donor mass in GRO J1655-40 is close to the lower limit $1.4{\rm M}_\odot$ found by Phillips et al. (1998) the transfer rate would be well below the critical rate for transient behaviour even in the absence of jet–induced CAML. However, the luminosity given by the spectral type and orbital period is too high for a $1.4{\rm M}_\odot$ donor in the phase of crossing the Hertzsprung gap (e.g. Kolb 1998). This suggests that the actual donor mass is nearer to the upper limit ($2.2{\rm M}_\odot$) quoted by Phillips et al. (1998). (We note that Regős et al. (1998) suggested that the donor in GRO J1655-40 could be still in the core hydrogen burning phase if the main sequence is significantly widened by convective overshooting in the star. This would also give a transfer rate smaller than the critical rate for any Γ).

The jet source SS433 could be affected by jet–induced CAML as well. The nature of the compact star in this system is still unclear (e.g. Zwitter & Calvani 1989; D'Odorico et al. 1991), but a black hole cannot be ruled out. The claimed mass ratio of order 3 is certainly above the stability limit for conservative mass transfer. However, the donor might not fill its Roche lobe (Brinkmann et al. 1989), and the jets seem to be less energetic than in the superluminal sources GRO J1655-40 and GRS 1915+105.

6 CONCLUSIONS

We may draw the following conclusions from the arguments of this paper:

1. The mass of a binary black hole changes by only a relatively small amount during the mass transfer process, whether through accretion (fractional increase $\lesssim M_2/M_1$ for a low–mass system, and much less for a high–mass system) or rotational energy extraction (fractional decrease $\lesssim 10\%$). Currently measured masses are therefore similar to the formation masses.

2. Too little mass is accreted to spin up a hole from $a_* \simeq 0$ to $a_* \simeq 1$.

3. Extracting angular momentum from the hole can in principle reduce $a_* \simeq 1$ to $a_* \simeq 0$, but in practice none of the known systems shows the effects of injecting the extracted rotational energy of about 10^{54} erg into the local ISM. In combination with 2. above this suggests that binary black holes also retain a value of a_* close to that at formation.

4. Jet ejection powered by the black hole's rotational energy can have a major effect in stabilizing mass transfer, particularly in higher–mass systems. Sufficiently energetic jets can reduce the mass transfer rate to values making the system transient.

If we accept 3. above, it would appear that any claimed range of a_* or M_1 must represent the range of initial conditions for binary black holes. However, a simple extrapolation of the neutron star case seems to favour values $a_* \simeq 1$. In particular even the maximum angular momentum $J_1 = GM_1c = 3\times10^{37}(M_1/10{\rm M}_\odot)$ cm^2 s for the hole is far smaller than any plausible value for that of the progenitor star. If confirmed, the range of a_* claimed by Zhang et al. (1997) thus represents a challenge to theory.

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Table 1. Binaries with claimed measurements of black–hole spin a_*.

system	orbital period	a_*	type	$M_{\rm tr}/{\rm M}_\odot$	$M_{\rm spin-up}/{\rm M}_\odot$
$GS1124-68$	10.4 h	-0.04	j–driven	< 1.5	-
$GS2000+25$	8.3 h	0.03	j–driven	< 1.5	-
LMC X-3	1.7 d	-0.03	n–driven	$\lesssim 2.5$	-
			wind–fed	$\lesssim 0.1$	
GRO J1655-40	2.62 d	0.93	n–driven	< 1.9	3.3
		> 0.7			1.9
GRS 1915+105	?	0.988?	n–driven	$\lesssim 5$	22
			wind–fed	$\lesssim 0.1$	

FIGURE CAPTIONS

Figure 1 *Top panel:* Estimated upper limit $M_{\rm acc}$ for the mass a black hole can accrete during case A mass transfer, as a function of initial donor mass $M_{2,\rm initial}$ (assuming that the donor is initially on the ZAMS and the black hole has mass $8{\rm M}_\odot$). *Middle panel:* Final period $P_{\rm f}$ (solid, scale on the left) and initial period $P_{\rm i}$ (dashed, scale on the right). *Bottom panel:* Total duration t of mass transfer phase (ending when the donor reaches the terminal main sequence; solid, scale on the left) and final secondary mass $M_{2,\rm f}$ (dashed, scale on the right). The donor's mass–radius index ζ was fixed at 0.

Figure 2 As Fig. 1, but for case B mass transfer, with the donor initially on the terminal main-sequence. Mass transfer terminates when core helium burning begins.

Figure 3 Black hole spin a_* vs. accreted rest–mass ΔM, in units the final mass $M_{\rm max}$ at maximum spin (solid line, bottom axis), and in units of the initial mass $M(a_* = 0)$ at zero spin (dashed, top axis).

Figure 4 Extracted fractional energy $\Delta m = (M_{\rm max} - M_1)/M_{\rm max}$ as a function of efficiency ϵ of extraction of a black hole's rotational energy, cf. (12).

Figure 5 Stability term D from eq. (20) vs. mass ratio $q = M_2/M_1$, for jets with different specific energy Γ (assuming $\zeta = 0$).

Figure 6 Mass transfer reduction factor $D(\Gamma)/D_0$ vs. mass ratio $q = M_2/M_1$, for jets with different specific energy Γ (assuming $\zeta = 0$).

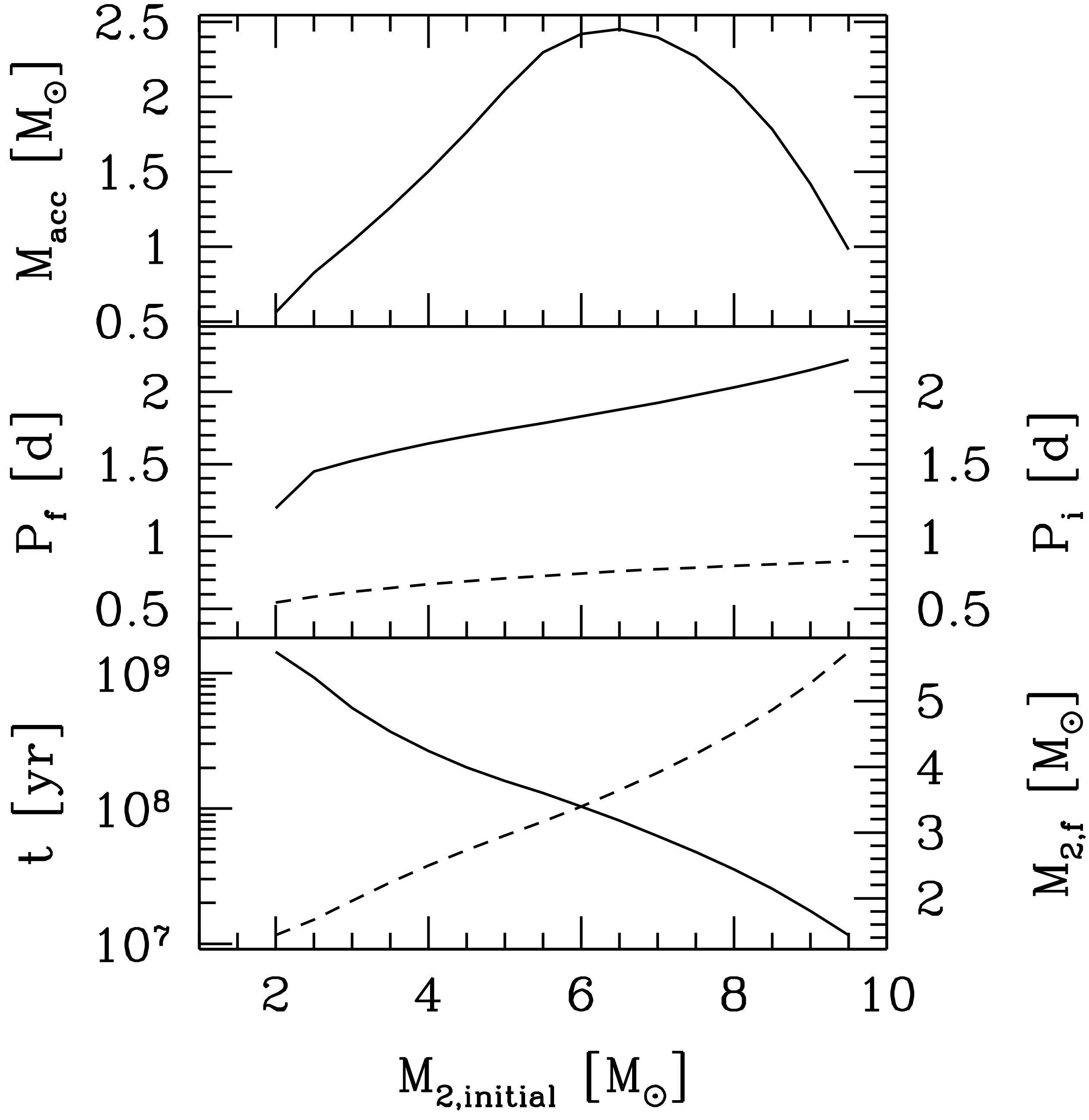
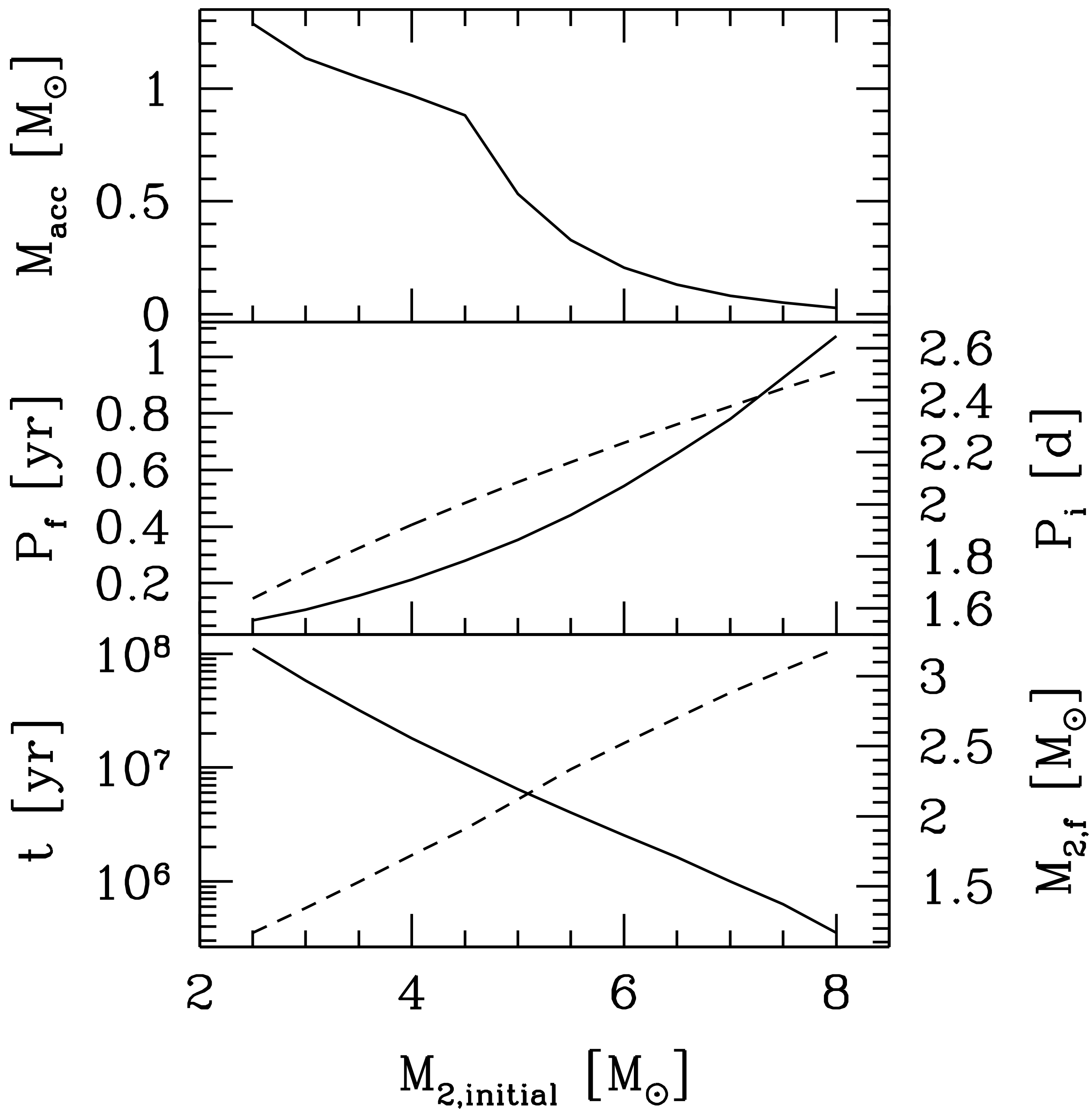

2.5
2
1.5
1
0.5
M_{acc} [$M_\odot$]
2
1.5
1
0.5
P_f [d]
2
1.5
1
0.5
P_i [d]
10^9
10^8
10^7
t [yr]
5
4
3
2
$M_{2,f}$ [$M_\odot$]
2
4
6
8
10
$M_{2,initial}$ [$M_\odot$]

Figure 1



M_acc [M_⊙]
0
0.5
1
P_f [yr]
0.2
0.4
0.6
0.8
1
P_i [d]
1.6
1.8
2
2.2
2.4
2.6
t [yr]
10^6
10^7
10^8
M_2,f [M_⊙]
1.5
2
2.5
3
2
4
6
8
M_2,initial [M_⊙]

Figure 2

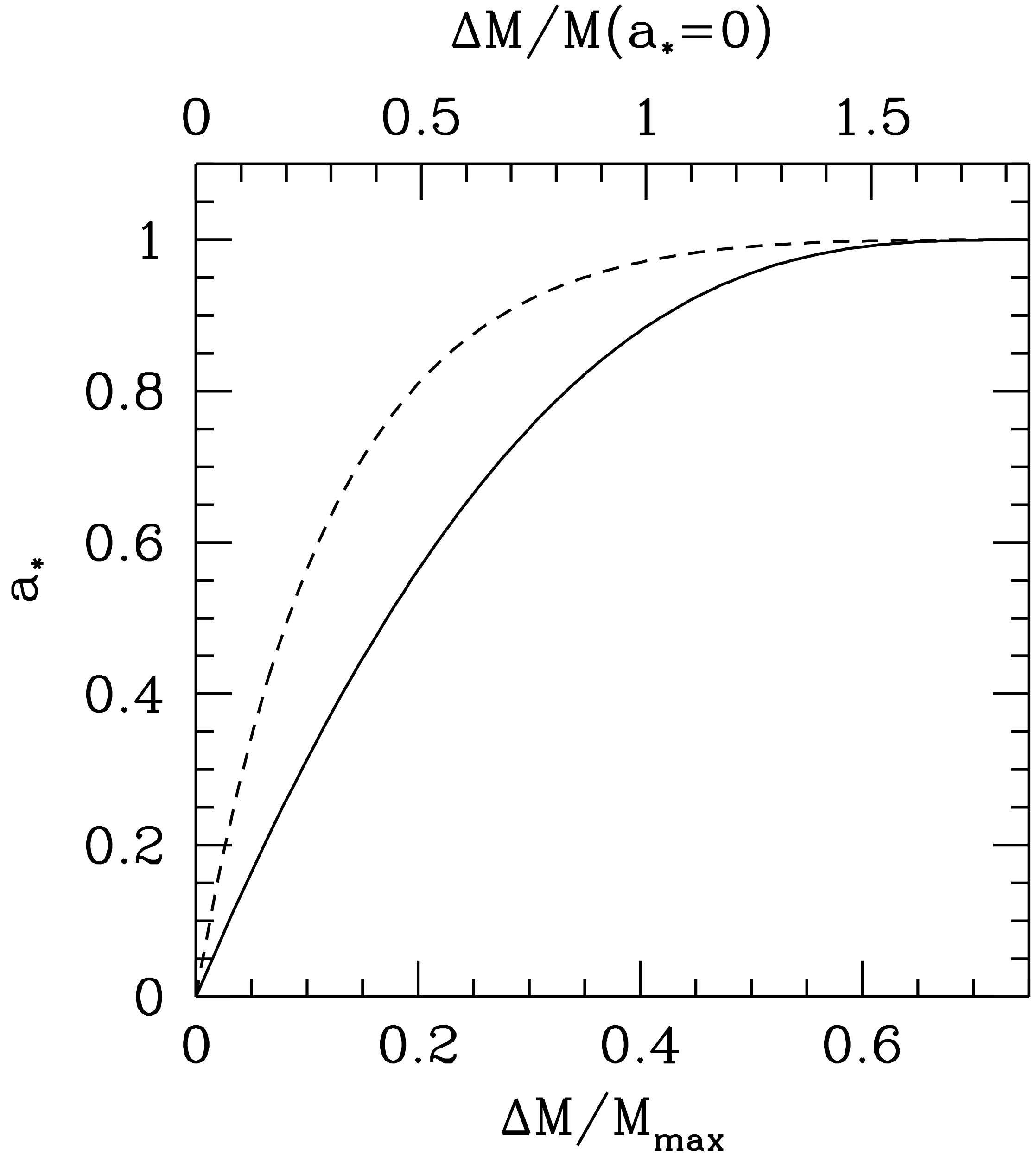


Figure 3

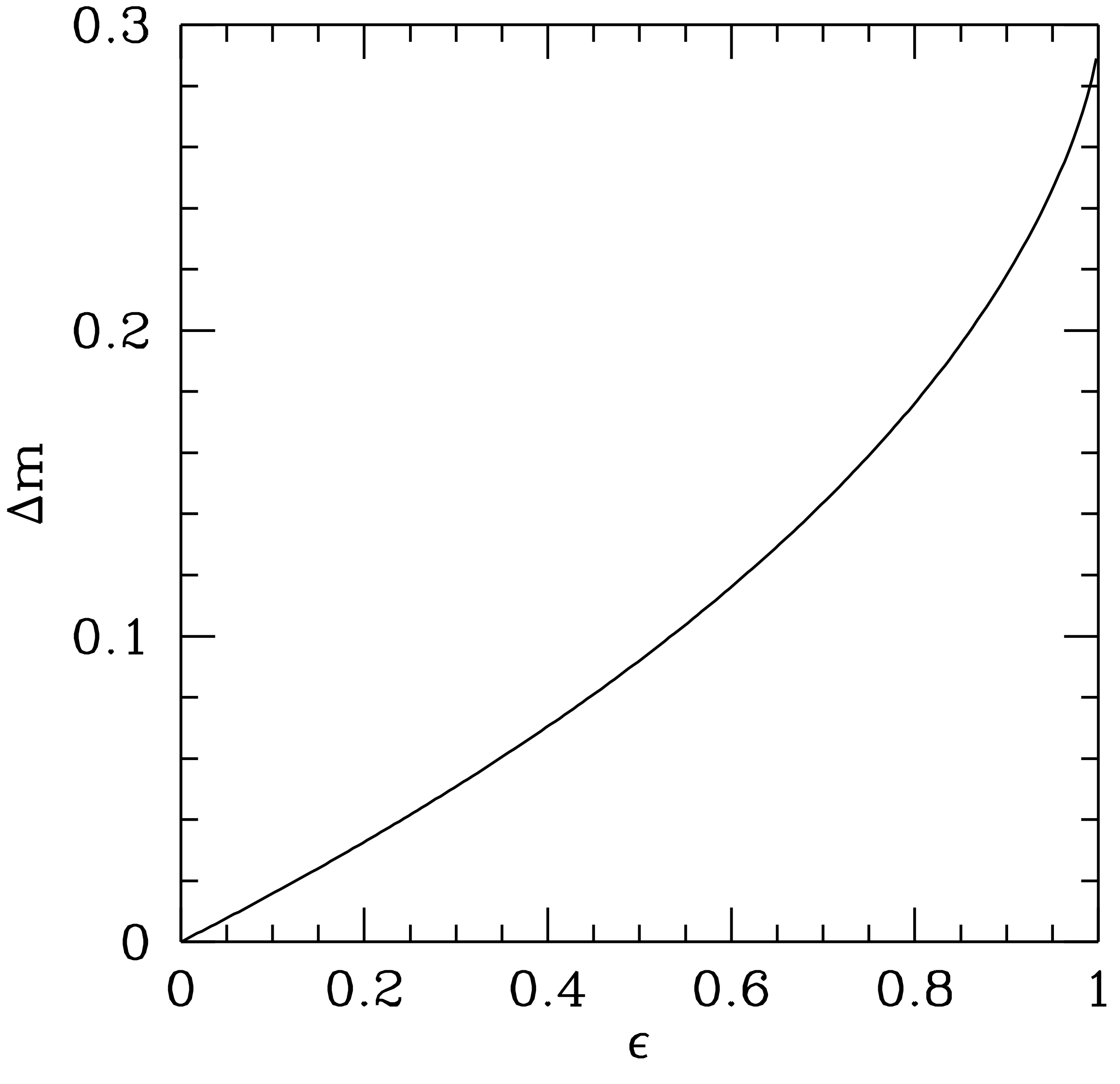


Figure 4

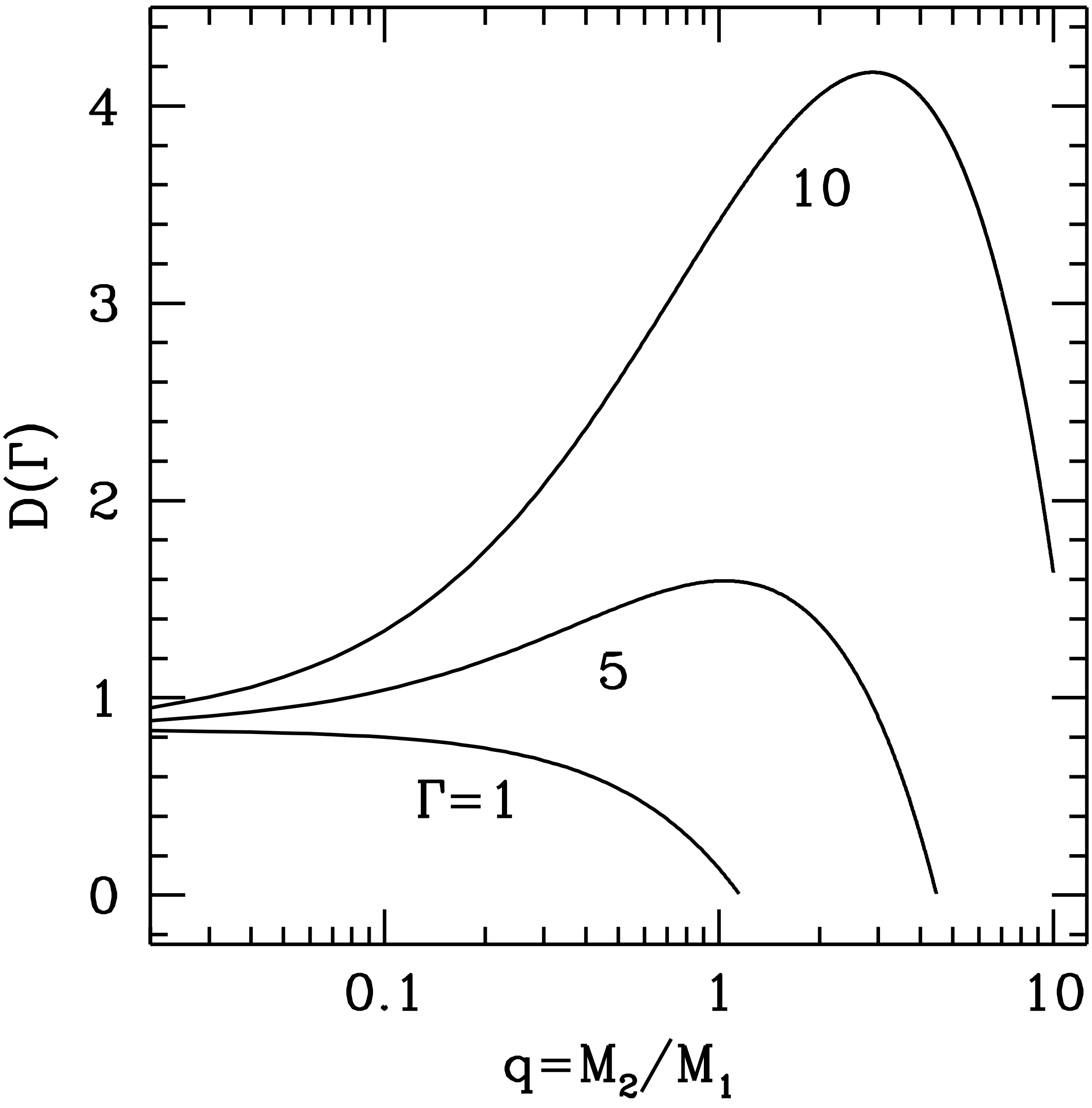


Figure 5

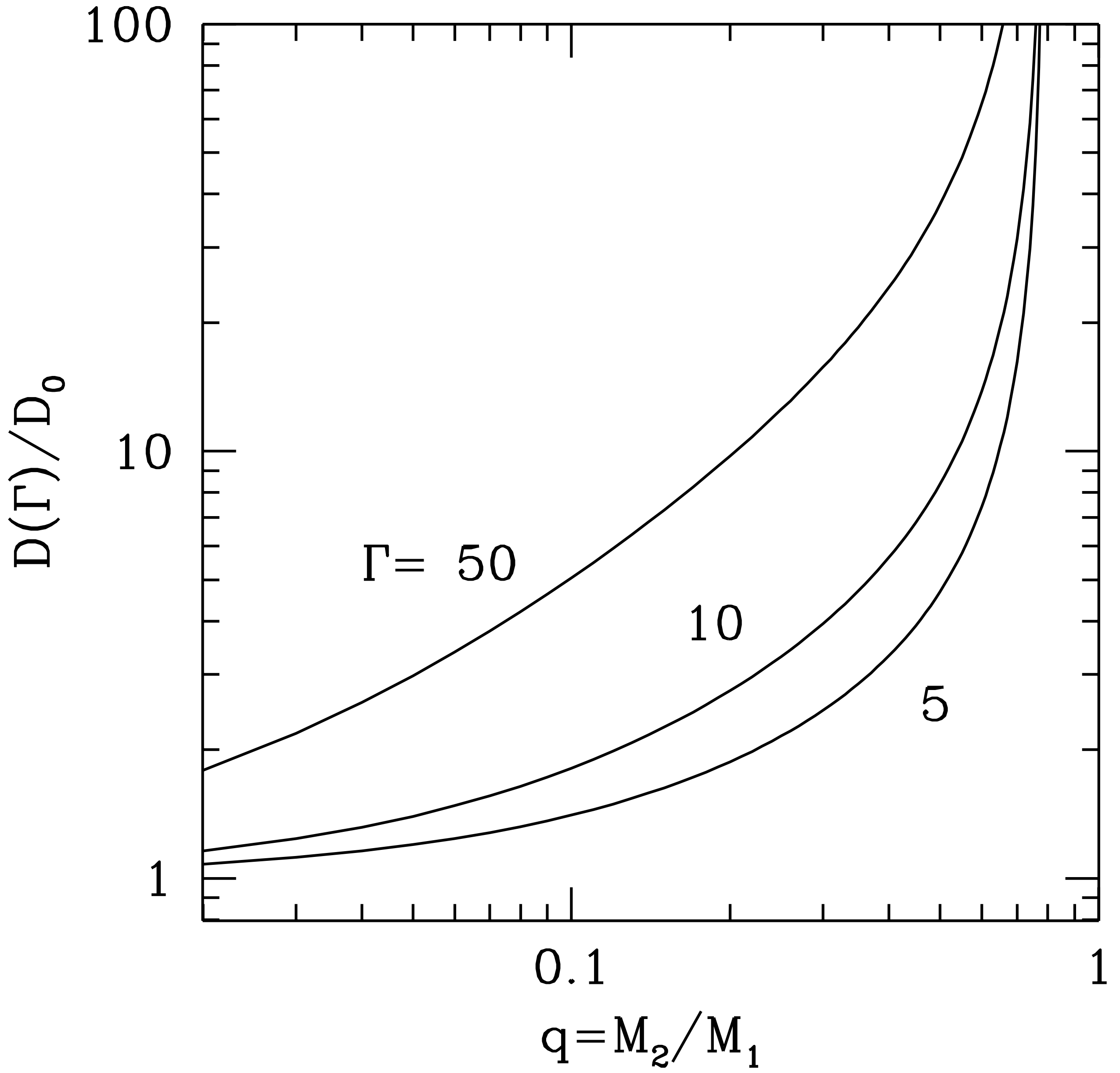


Figure 6