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The CV period minimum

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Abstract. Using improved, up–to–date stellar input physics tested against observations of low–mass stars and brown dwarfs we calculate the secular evolution of low–mass donor CVs, including those which form with a brown dwarf donor star. Our models confirm the mismatch between the calculated minimum period ($P_{\rm min} \simeq 70$ min) and the observed short–period cut–off ($\simeq 80$ min) in the CV period histogram. Theoretical period distributions synthesized from our model sequences always show an accumulation of systems at the minimum period, a feature absent in the observed distribution. We suggest that non–magnetic CVs become unobservable as they are effectively trapped in permanent quiescence before they reach $P_{\rm min}$, and that small–number statistics may hide the period spike for magnetic CVs. We calculate the minimum period for high mass transfer rate sequences and discuss the relevance of these for explaining the location of CV secondaries in the orbital period - spectral type diagram. We also show that a recently suggested revised mass–radius relation for low–mass main–sequence stars cannot explain the CV period gap.

1. Introduction

We consider cataclysmic variables (CVs) at period–bounce, the evolutionary phase where the secular mean orbital period derivative changes from negative to positive. For predominantely convective donor stars (with masses $\lesssim 0.6 - 0.7 M_\odot$) period bounce occurs when the mass transfer timescale $t_M = M_2/(-\dot{M}_2)$ becomes small compared to the secondary's thermal time $t_{\rm KH} = GM_2^2/R_2L_2 \simeq 3 \times 10^7 {\rm yr}/(M_2/M_\odot)^3$. These stars expand and become less dense on rapid mass loss, in contrast to the increase of mean density $\rho = M_2/R_2^3$ towards smaller mass along the main sequence. When ρ decreases the orbital period becomes longer as $P \propto \rho^{-1/2}$ (from Roche geometry and Kepler's law).

The short–period cut–off of the CV orbital period distribution at 80 min, the "minimum period", has long been interpreted as a consequence of period bounce (Paczyński & Sienkiewicz 1981; Rappaport, Joss & Webbink 1982) coinciding with the donor's transition from a main–sequence star to a brown dwarf (BD). Period bounce occurs at longer orbital period and correspondingly higher donor mass if the mass transfer rate is much higher than the rate driven by gravitational wave emission. CVs are close to period bounce at the upper edge of the CV period gap for transfer rates required to fit the width and location of the gap within the standard period gap model. Yet higher rates and corresponding bounce at still longer P may be required to explain the observed location of certain CVs in the orbital period - spectral type diagram (Beuermann et al. 1998).

Here we investigate the behaviour of CVs at period bounce by applying up–to–date stellar models for low–mass stars and brown dwarfs (Baraffe et al. 1995, 1997, 1998; henceforth summarized as BCAH) which reproduce observed properties of field M–dwarfs with unprecedented accuracy. Main features of these models are the improved internal physics (Chabrier and Baraffe 1997) and outer boundary conditions based on non-grey *NextGen* atmosphere models and synthetic spectra of Hauschildt et al. (1998; see also Allard et al. 1997).

2. The minimum period at 80 min

The observed period distribution of hydrogen–rich donor CVs shows a sharp cut–off at $P \simeq 80$ min (e.g. Ritter & Kolb 1998), the short end being marked by J0132–6554 at $P = 77.8$ min. The notable exception is V485 Cen at $P = 59$ min, while six AM CVn type CVs with yet shorter periods are interpreted as helium–star CVs.

2.1. Model calculations

Theoretical period distributions calculated by population synthesis techniques (e.g. Kolb 1993, 1996; Howell, Rappaport & Politano 1997) so far fail to reproduce the location of the minimum period $P_{\rm min}$ (they typically give a cut–off at $60 - 65$ min), and predict a significant accumulation of systems near $P_{\rm min}$, a "period spike", which is not observed. The spike is caused by the fact that the detection probability d per period bin is inversely proportional to the velocity $\dot{P}$ in period space, hence $d \to \infty$ at period bounce $\dot{P} = 0$.

The population models rely on a number of simplifications. The donor is usually approximated as a polytrope, with a surface boundary condition calibrated to full stellar models. In the case of Kolb (1993) these were obtained using Mazzitelli's stellar code as of 1989 (e.g., Mazzitelli 1989), with now partly out–dated stellar input physics. Equally important, CVs forming with donor mass $M_2 \lesssim 0.1 M_\odot$ are not considered. All CVs with smaller M_2 in the population models descended from systems with initially higher–mass donors.

To improve on this we recalculated the secular evolution of short–period CVs using the BCAH stellar evolution code. We focussed on CVs below the period gap and assumed that orbital angular momentum is lost by gravitational wave emission (e.g. Landau & Lifschitz 1952) and via an isotropic stellar wind from the white dwarf (WD), removing the accreted mass with the WD's specific

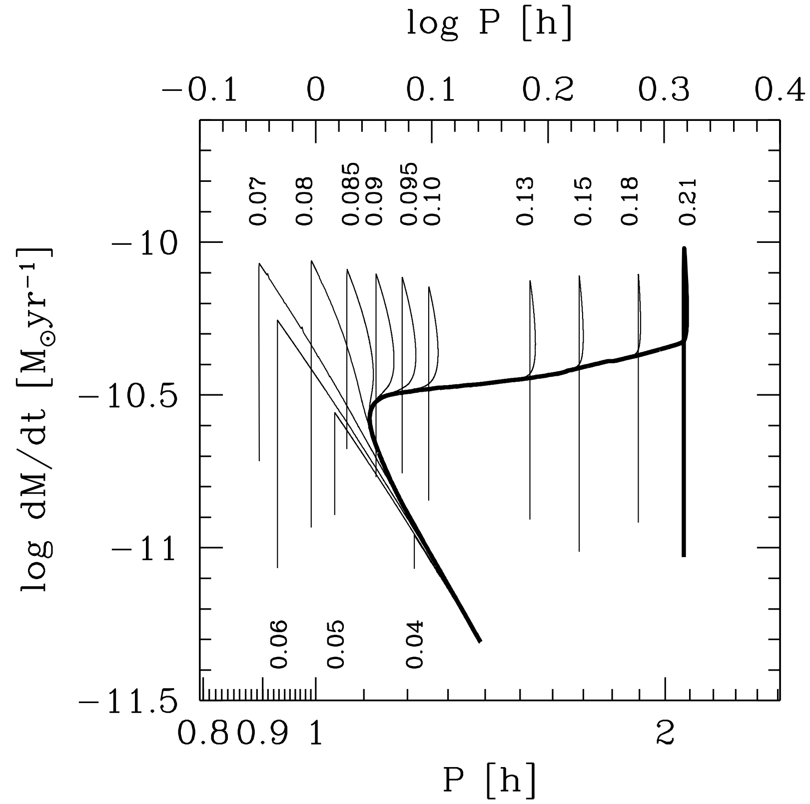

Figure 1. Mass transfer rate versus orbital period (left) and donor's effective temperature versus period (right) along evolutionary sequences for CVs with a $0.6M_\odot$ WD. The tracks are labelled with the initial donor mass (in $M_\odot$). The $0.21M_\odot$ sequence is shown in bold.

orbital angular momentum from the binary. With this choice the results do not depend on the rather uncertain strength of magnetic braking thought to dominate the evolution of CVs above the period gap (see e.g. King 1988 for a review).

In Fig. 1 we plot mass transfer rate versus period for a set of sequences with $0.6M_\odot$ WD mass and initial donor masses ranging from $M_2 = 0.04M_\odot$ to $0.21M_\odot$. Within the standard period gap model (e.g. Kolb 1996) the upper value is the donor mass at which CVs emerge from the detached phase that is responsible for the CV period gap. At turn–on of mass transfer the secondary was either on the ZAMS ($M_2 > 0.09M_\odot$), or had an age of 2 Gyrs ($M_2 \leq 0.09M_\odot$).

The figure confirms the well known effect that systems with different initial donor mass join rather quickly a uniform evolutionary track (Stehle et al. 1996). Most systems undergo period bounce at $P_{\rm min} \simeq 67$ min, which is only slightly longer than the corresponding $P_{\rm min} = 65$ min found with Mazzitelli's models (Kolb & Ritter 1992). Significantly, CVs forming with fairly old and massive

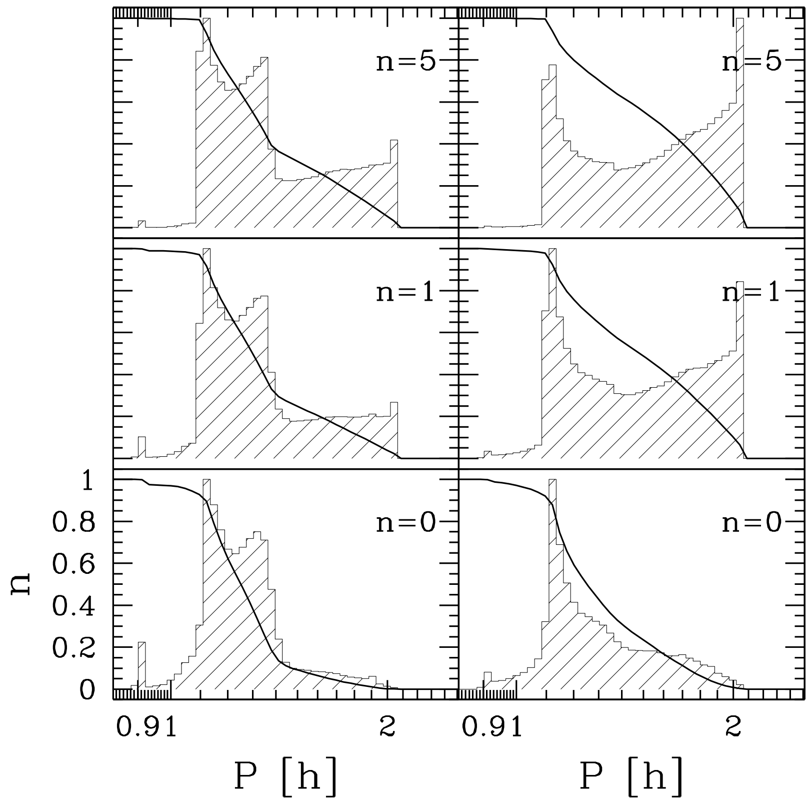

Figure 2. Theoretical orbital period distribution (histogram and cumulative distribution) for CVs with $0.6 M_\odot$ WD mass and donor mass $\leq 0.21 M_\odot$ (i.e. period $P < 2.1$ h), for different contributions n from systems born above the period gap. Left: volume–limited sample ($\alpha = 0$). Right: magnitude–limited sample ($\alpha = 1.5$). See text for details.

brown dwarf donors (age $\gtrsim 2$ Gyr, mass $0.05 - 0.07 M_\odot$) would populate the period regime shortwards of $P_{\rm min}$. V485 Cen could be such a CV.

The importance of CVs forming with BD donors for the observed period histogram depends on their relative formation rate. Here we consider the simple case of a constant birth rate b per logarithmic mass interval ($\partial b / \partial \log M_2 = 0$) for $0.04 \leq M_2/M_\odot \leq 0.21$. In the case of CVs with main–sequence secondaries this choice is roughly consistent with results from detailed calculations of CV formation via the standard common envelope channel (e.g. Politano 1996, de Kool 1992), while it is just an assumption for degenrate secondaries. (A more detailed appraisal of the BD CV formation rate is in preparation.) CVs which have formed above the period gap and evolved to short periods are taken into account by adding the contribution $n \times I$ to the birth rate in the first mass bin at $0.21 M_\odot$. This corresponds to a period $P \simeq 2.1$ h, so that the predicted distribution only applies to periods less than this. Here I is the total formation rate of CVs below the period gap and n a free parameter. Detailed standard models indicate $n \simeq 1$.

The period distribution for the subset of CVs with $0.6 M_{\odot}$ WD mass obtained from a convolution of the sequences in Fig. 1 with the above simple CV formation rate is shown in Fig. 2, for $n = 0$, 1 and 5. The left panel shows the distribution of a volume–limited sample, the right panel a sample where individual systems have been weighted by $\dot{M}^{\alpha}$, with $\alpha = 1.5$ ($\dot{M}$ is the mass transfer rate). With this brightness–dependent factor the period distributions mimic those expected for $m_{\rm vis}$– or $m_{\rm bol}$–limited samples. The proper value for the parameter α is of order unity, but depends in detail on the distribution of objects in physical space and the emission properties of accretion discs with accretion rate $\dot{M}$ (cf. Kolb 1996). The population shown in the figure formally corresponds to an age of 6 Gyr for the Galactic disc; in a somewhat older population the edge of the volume–limited distributions at $\simeq 1.4$ hr would appear at slightly longer P.

Figure 2 shows that for the adopted formation rate BD CVs would not contribute significantly to the period distribution in a magnitude–limited sample. In particular, BD CVs have no effect on the overall shape of the period spike at $P_{\rm min}$. They would have an effect only if b were strongly increasing towards small masses. In this case the short–period end of the distribution would be at even shorter periods, clearly in conflict with observations.

The overall shape of the weighted distribution is not sensitive to α; there is hardly any difference between the cases with $\alpha = 1$ and $\alpha = 1.5$. Increasing α tends to decrease the amplitude of the spike, but we obtain a distribution which is roughly flat – similar to the observed one – only for $\alpha \gtrsim 6$. This holds also for calculated period distributions which take into account the full WD mass spectrum expected for CVs (see Kolb & Baraffe 1998 for an example obtained with deKool's (1992) CV formation rate).

In summary: suppressing the period spike in the distribution requires a very steep dependence of the detectability on $\dot{M}$.

2.2. Interpretation

Essentially all known short–period non–magnetic CVs are dwarf novae, with similar absolute magnitudes in outburst (e.g. Warner 1995). A few of them, sometimes referred to as WZ Sge stars, have very long outburst recurrence times $t_{\rm rec}$, the most extreme example being WZ Sge itself with $t_{\rm rec} \simeq 30$ yr. It has long been noted that low–$\dot{M}$ CVs might have escaped detection if their outburst interval is significantly longer than the period since beginning of systematic monitoring and surveying of the sky with modern means – a few decades.

For long $t_{\rm rec}$ the relative detection probability of dwarf novae scales as $d \propto 1/t_{\rm rec}$, suggesting that $t_{\rm rec} \propto \dot{M}^{-\alpha} \propto \dot{M}^{-6}$ or steeper. In practice this very steep dependence would require that $t_{\rm rec} \to \infty$ for $\dot{M} \lesssim$ few $\times 10^{-11} M_{\odot} {\rm yr}^{-1}$, i.e. low–$\dot{M}$ CVs would not undergo outbursts at all.

A plausible physical model which naturally accounts for this property is the extreme version of the evaporating accretion disc model suggested for WZ Sge by Meyer–Hofmeister et al. (1998; see also Liu et al. 1997). Evaporation of accreted material into a hot corona could prevent the disc from accumulating the critical surface mass density required to launch a heating wave. Systems in

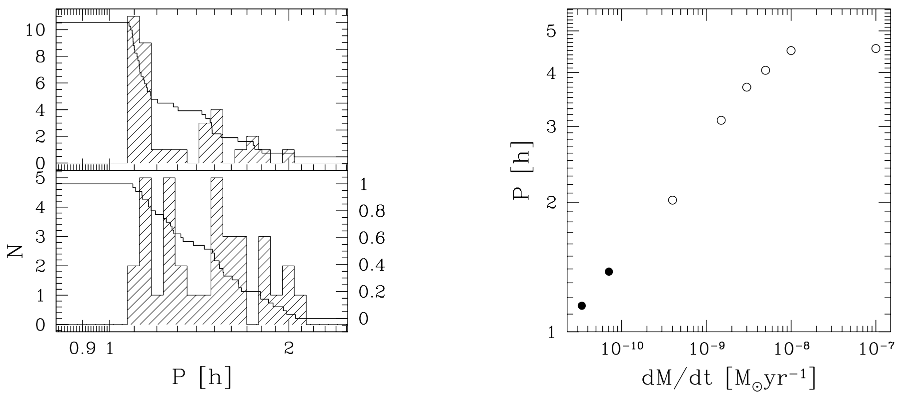

Figure 3. *Left:* Two representative samples of 35 systems drawn from the parent distribution shown in Fig. 2, middle right. *Right:* Orbital period at period bounce versus mass transfer rate at this point. Solid dots represent sequences calculated with angular momentum loss $\dot{J} = \dot{J}_{\rm GR}$ and $4 \times \dot{J}_{\rm GR}$, open dots are sequences calculated with a constant mass transfer rate.

this permanent quiescence are optically very faint but should emit about 10% of their accretion luminosity $L_{\rm acc} = GM_1\dot{M}/R_1$ (M_1, R_1 is the WD's mass and radius) in X–rays. See Watson (this volume) for a plot of the corresponding intrinsic X–ray luminosity function of a typical Galactic CV population (model dK1in of Kolb 1993), and for a discussion of the detectability of such an X–ray background from low–luminosity CVs and its potential contribution to the Galactic ridge emission.

Although this could explain the non–detection of the period spike at $P_{\rm min}$ for dwarf novae, it certainly would not apply to discless systems, i.e. for the $\simeq 35$ polars with $P \lesssim 2$ h (Beuermann 1997; Ritter & Kolb 1998). An independent observational selection effect operating on polars in the same period/mass transfer rate range and with the same net result as a steep increase of $t_{\rm rec}$ for dwarf novae seems highly unlikely. In other words: polars should show a period spike, even though dwarf novae do not. Small number statistics could be responsible for the non–detection of the polar spike. Monte Carlo experiments where a sample of 35 systems is drawn from an underlying period distribution like the one shown in Fig. 2 (middle right) give a surprisingly wide variety of distributions (Fig. 3, left), with many of them showing no sign of a period spike at all. In contrast, in a sample with $\gtrsim 80$ systems (dwarf novae with $P < 2.1$h) the spike is almost always prominent.

This explanation for the missing period spike allows the theoretically computed value of $P_{\rm min}$ to be shorter than the observed one. Nelson et al. (1985) found that their low–mass stellar models increase the computed minimum period by $\simeq 10\%$ when tidal and rotational corrections are applied to the 1–dim. stellar structure equations (Chan & Chau 1979). Preliminary calculations based on the BCAH models do not confirm this result; we find a rather modest increase of $\simeq 1\%$ (cf. Kolb and Baraffe 1998 for details). While we do not claim that the true theoretical value of $P_{\rm min}$ must be longer than what our models give, there are indeed uncertainties in the calculated value of $P_{\rm min}$ inherent to the very concept of the Roche model. Strictly valid only for point masses, its applicability to extended stars relies on the fact that stars are usually sufficiently centrally condensed. This is not necessarily a good approximation for fully convective stars which are essentially polytropes of index 3/2.

Finally, we note that the predicted $P_{\rm min}$ is longer if the orbital angular momentum losses $\dot{J}$ are larger than the value $\dot{J}_{\rm GR}$ from gravitational radiation. We find $P_{\rm min} \simeq 83$ min (up from 69 min) for $\dot{J} = 4 \times \dot{J}_{\rm GR}$ and $M_1 = 1 M_\odot$, a much smaller increase than quoted by Paczyński (1981). Patterson (1998) favoured a modest increase of $\dot{J}$ over $\dot{J}_{\rm GR}$ on grounds of space density considerations and the number ratio of CVs above/below the gap. However, postulating an as yet unknown $\dot{J}$ mechanism which conspires to produce almost the same value as $\dot{J}_{\rm GR}$ at the transition from non–degenerate to degenerate stars does not seem very attractive.

A more thorough account of the discussions in this section is given in Kolb & Baraffe (1998).

3. Period bounce at longer periods

Reversal of the orbital period derivative is not uniquely confined to the transition region between main–sequence and degenerate stars. Period bounce occurs at longer periods and higher donor masses if the system has a higher mass transfer rate than the one driven by gravitational radiation. As an example, we show in Fig. 3 (right) how $P_{\rm min}$ varies with the transfer rate.

The potential importance of evolutionary sequences with fairly high mass transfer rates become clear from Fig. 4 (right) where we plot the donor's spectral type versus orbital period (data taken from Beuermann et al. 1998) and a "standard" evolutionary sequence that reproduces the width and location of the period gap (2.1 – 3.2 h). This standard sequence was calculated with constant mass transfer rate $1.5 \times 10^{-9} M_\odot {\rm yr}^{-1}$ until the secondary became fully convective, when mass loss was terminated and the star allowed to shrink back to its equilibrium radius. Mass loss resumed (at the now shorter orbital period $P = 2.1$ hr) with a rate $5 \times 10^{-11} M_\odot$ yr^{-1}, typical for mass transfer driven by gravitational wave emission.

Although the BCAH models successfully reproduce properties of low–mass single stars (Baraffe & Chabrier 1996, Baraffe et al. 1997, 1998, and references therein) the standard sequence clearly fails to give the observed late spectral types for a given period above the gap. As has been shown by Beuermann et al. (1998) CVs with donors that are somewhat evolved off the ZAMS (but still in the core hydrogen–burning phase) can account for the late spectral types in long–period systems ($P \gtrsim 6$ h), but not in those close to the upper edge of the gap ($3 \lesssim P/{\rm h} \lesssim 6$) where evolved and unevolved sequences merge. (Further details will be presented in Baraffe & Kolb 1998).

Generally, low–mass stars in the phase of core hydrogen burning subjected to mass loss become over- or underluminous compared to stars in thermal equilibrium, but in such a way that the effective temperature hardly changes (King & Kolb 1998). Hence the effective temperature, or the spectral type, is an indicator of the stellar mass, whatever the previous mass–loss history.

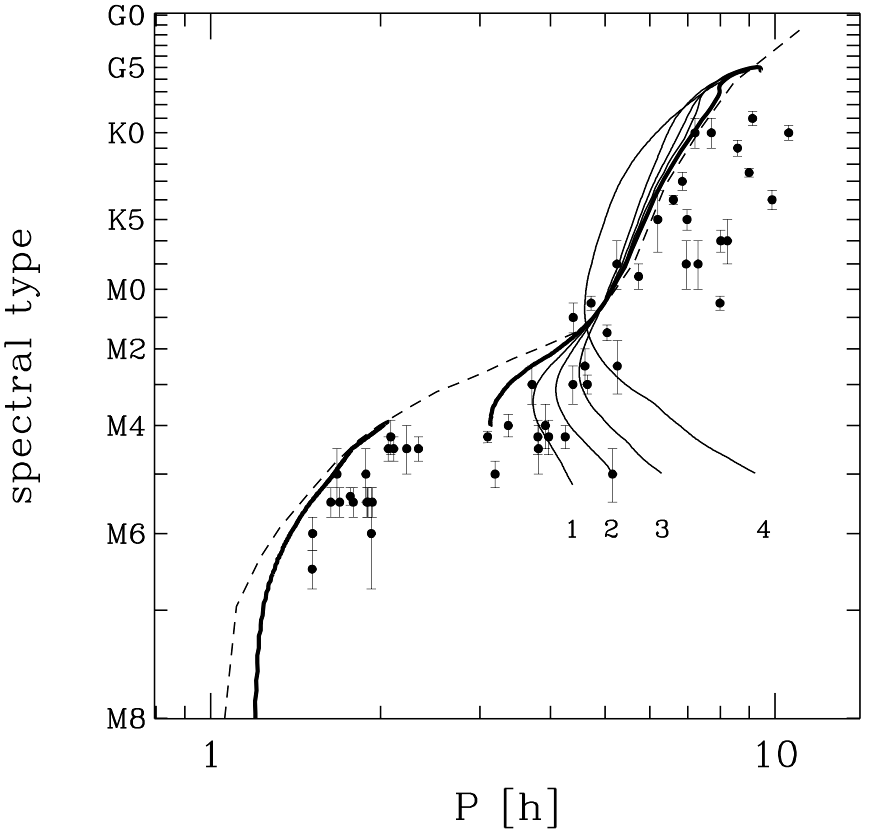

Figure 4. Spectral type of the secondary versus orbital period. Data points from Beuermann et al. 1998; dashed: ZAMS; heavy solid line: standard sequence (see text); solid: sequences with constant mass loss rate (1: $3 \times 10^{-9} M_\odot \mathrm{yr}^{-1}$; 2: $5 \times 10^{-9} M_\odot \mathrm{yr}^{-1}$; 3: $10^{-8} M_\odot \mathrm{yr}^{-1}$; 4: $10^{-7} M_\odot \mathrm{yr}^{-1}$).

If the observed late spectral types of CV secondaries for $P \lesssim 6$ h are a consequence of mass transfer alone, then these secondaries must be severely undermassive, i.e. less massive than a main–sequence star that would fill its Roche lobe at the same period. As the degree of "undermassiveness" increases with mass transfer rate $\dot{M}$, we conclude that in systems with $3 \lesssim P/\mathrm{h} \lesssim 6$ and cool donors, $\dot{M}$ must be higher than in the standard sequence. As an extreme example, AR Cnc with spectral type M5 and $P = 5.15$ h would fit on a track with $\dot{M} = 5 \times 10^{-9} M_\odot \mathrm{yr}^{-1}$, at a donor mass $\simeq 0.15 M_\odot$. (We note that this transfer rate is only marginally consistent with the fact that AR Cnc is a dwarf nova). With such a small donor mass and high transfer rate this system does not fit into the framework of the standard model of the period gap, where the orbital decay rate drops sharply at $M_2 \simeq 0.2 M_\odot$. If a significant fraction of systems close to the upper edge of the gap were similar to AR Cnc, the period gap would not show up in the collective period distribution. To verify if this is indeed a problem for the standard CV period gap model, an observational determination of the donor mass in systems like AR Cnc is vital.

4. Main–sequence mass–radius relation and CV period gap

As highlighted in the previous section the spectral types of certain CV secondaries, when interpreted as a result of their mass loss history, may be in conflict with the standard CV period gap model. Numerous alternative explanations for the period gap have been put forward in the literature, but none of them has proved to be as successful as the disrupted orbital decay model. The most recent alternative suggestion, by Clemens et al. (1998), claims that a characteristic feature of the low–mass main–sequence mass–radius relation $R(M)$ would translate into a rapid evolution through the period gap region even for continuous angular momentum losses, reducing the discovery probability there. In particular, the logarithmic slope $\zeta = \mathrm{d}\ln R/\mathrm{d}\ln M$ of Clemens et al.'s observationally derived $R(M)$ relation shows two discontinuities. With decreasing mass ζ jumps from a rather large value $\gtrsim 1$ to a rather small $\zeta \simeq 0.33$ at mass $\simeq 0.5 M_\odot$ and $\simeq 0.2 M_\odot$. A corresponding structure may already be evident in the underlying colour–magnitude diagram, with disputable significance. Note that theoretical models, which now reach a good agreement with observed stellar parameters, e.g. the eclipsing binary CM Draconis (Chabrier and Baraffe 1995) or the data of Leggett et al. 1996, do not predict such a feature in the $R(M)$ relationship of low-mass stars.

If mass transfer in CVs proceeds sufficiently slowly (as assumed by Clemens et al. 1998), the donor star's radius simply follows the main–sequence $R(M)$ relation. As

$$\frac{\dot P}{P} = \frac{3}{2}\frac{\dot R_2}{R_2} - \frac{1}{3}\frac{\dot M_2}{M_2} = \frac{1}{2}\left(3\zeta - 1\right)\frac{\dot M_2}{M_2}, \tag{1}$$

the discovery probabilty $d(\log P) \propto \dot M/(\dot P/P)$ in a magnitude–limited sample is $d \propto 1/(3\zeta - 1)$. Hence the $R(M)$ relation claimed by Clemens et al. would produce a pair of period "spikes" at 3.4 hr and 2.0 hr where $\zeta \simeq 1/3$ ($d \to \infty$). (The CVs are actually close to bounce at these periods, and the spikes appear for the same reason as the spike at $P_{\rm min}$ discussed in Sect. 1.) The discovery probability between the two spikes is no lower than outside them, there is no period "gap". This has been verified with a full population synthesis calculation, adopting continuous, weak orbital decay by gravitational radiation only, with the claimed $R(M)$ relation imposed on lower main–sequence stars (Kolb et al. 1998).

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