Crandall memorial puzzle

In honour of the memory of Richard Crandall (1947–2012), I have devised a puzzle on prime numbers obtained from moments of Bessel functions.

For positive integer n, let

$$A(n) \equiv \left(\frac{2}{\pi}\right)^4 \int_0^\infty \left([\pi I_0(t)]^2 - [K_0(t)]^2\right) I_0(t)[K_0(t)]^5 (2t)^{2n-1} dt$$

where $I_0(t)$ and $K_0(t)$ are Bessel functions. Anton Mellit and I conjecture that $A(n)$ is a sequence of non-negative integers, beginning with
`0, 1, 2, 15, 302, 12559, 900288, 98986140, 15459635718.`

The integers $A(n)$ are rich in odd prime factors $p < n/2$. For example,

$$\begin{aligned} A(33) &= 2 \times 3^{23} \times 5^{11} \times 7^6 \times 11^2 \times 13^2 \times p_{56} \\ A(36) &= 3^{25} \times 5^{13} \times 7^8 \times 11^4 \times 13^2 \times 17^2 \times p_{59} \\ A(49) &= 2^3 \times 3^{39} \times 5^{16} \times 7^{10} \times 11^6 \times 13^4 \times 17^2 \times 19^3 \times 23^2 \times p_{86} \end{aligned}$$

where p_{56} is the 56-digit prime
`57992474894877287439798522082574263282518819530344295461.`
Similarly, p_{59} and p_{86} are primes with 59 and 86 decimal digits.

Consider the sequence of integers n for which $A(n)$ has precisely one prime divisor $p \geq n/2$. It begins with `3, 5, 7, 9, 11, 33, 36, 49,` and contains 5 more integers with $n \leq 2000$. There are 7 more integers $n \leq 60000$ for which $A(n)$ has only one probable prime divisor $p \geq n/2$.

Puzzle: How many of these 12 integers $n \in [50, 60000]$ can you find?

David Broadhurst, 4 October 2015