The knottiest of 9-crossing knots is 9_{33}

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Let $[r, s]$ denote the signature and $n := r + 2s$ the degree of the invariant trace field of a hyperbolic knot $\mathcal{K}$. Let p be the largest prime in the discriminant D of this algebraic number field.

I have computed these arithmetic knot invariants up to 9 crossings. The discriminants of 9_{32} and 9_{33} were obtained with David Bailey's implementation of PSLQ, working at 512-digit precision; the remainder with Pari's LLL, invoked by Snap at no more than 150-digit precision. Factors of the discriminant of 9_{30} were obtained with Richard Crandall's Giants; the remainder with Maple.

The only knots up to 9 crossings that are devoid of symmetry are 9_{32} and 9_{33}. Computationally speaking, they are the knottiest pair. By the methods of http://xxx.lanl.gov/abs/hep-th/9811173 their hyperbolic volumes were computed to 1850 digits. I accord the distinction of knottiest of all to 9_{33}, with the 36-digit prime 174738620136975814197184218325839257 in its 40-digit discriminant.

PS: all non-alternating 10-crossing knots now done, with 10_{149} and 10_{151} analyzed by PSLQ.

Table 1: Arithmetic invariants below 9 crossings.

$\mathcal{K}$	r	s	n	D	p
4_1	0	1	2	−3	3
5_2	1	1	3	−23	23
6_1	0	2	4	257	257
6_2	1	2	5	1777	1777
6_3	0	3	6	−10571	31
7_2	1	2	5	4409	4409
7_3	2	2	6	78301	78301
7_4	1	1	3	−59	59
7_5	2	3	8	−4690927	10589
7_6	1	4	9	90320393	649787
7_7	0	2	4	257	257
8_1	0	3	6	−92051	92051
8_2	2	3	8	−21309911	11317
8_3	0	4	8	60020897	1879
8_4	1	4	9	1160970913	683
8_5	1	2	5	8968	59
8_6	1	5	11	−303291012439	596831
8_7	1	5	11	−121118604943	518867
8_8	0	6	12	2885199252305	1843577797
8_9	0	6	12	421901335721	281
8_{10}	1	5	11	−170828814392	21353601799
8_{11}	0	5	10	−2334727687	2423
8_{12}	0	7	14	−15441795725579	511591
8_{13}	0	7	14	−759929100364387	94430797
8_{14}	1	7	15	−26196407237223439	10310203
8_{15}	1	3	7	−1172888	541
8_{16}	1	2	5	5501	5501
8_{17}	0	9	18	−25277271113745568723	18013
8_{18}	2	1	4	−448	7
8_{20}	1	2	5	5864	733
8_{21}	0	2	4	392	7

Table 2: Arithmetic invariants at 9 crossings.

$\mathcal{K}$	r	s	n	D	p
9_2	1	3	7	−2518351	1279
9_3	3	3	9	−2452976839	57045973
9_4	2	4	10	105672896309	465519367
9_5	1	5	11	−2835434699687	2835434699687
9_6	2	5	12	−5761111009127	1130627
9_7	2	6	14	20663665389554981	62376545597
9_8	1	7	15	−388094531402024143	903211277621
9_9	3	6	15	306436083467784929	782895769
9_{10}	2	2	6	182977	2731
9_{11}	2	7	16	−65086143976777458247	32971
9_{12}	1	8	17	731484325942209432753	156953399
9_{13}	2	8	18	5701677528333538961 1741	107055662485843499 9
9_{14}	0	9	18	−20795318287129449084691	1182223893526404 1549
9_{15}	1	9	19	−5251019930631086958 43751	705354674871964301
9_{16}	2	3	8	−61495736	599
9_{17}	1	3	7	−1077683	9887
9_{18}	2	9	20	−190439786903825633 8349663	597308242335494256 61
9_{19}	0	10	20	1079496105055146923 5784881	7864587479501
9_{20}	2	9	20	−338073515883183637 94155399	173923018174549613
9_{21}	1	10	21	149893136961666539311912297	25236662196853
9_{22}	1	11	23	−53275843189911955342445943 57067	84430813296215460130659 18157
9_{23}	1	2	5	5653	5653
9_{24}	1	8	17	481202274808835122 96	432779167693
9_{25}	1	12	25	40556054799556962784805443827 62333	244312251595856996 33
9_{26}	1	11	23	−39635608642982347623217134 5159	1322785515678761
9_{27}	0	12	24	16547841469848921884806617976601	2418154422252199
9_{28}	0	2	4	117	13
9_{29}	2	4	10	127216336673	219463
9_{30}	0	14	28	22071188909526930809658014743039031 3113	5918911898147470846 3
9_{31}	1	3	7	−913451	11863
9_{32}	1	14	29	65771196430462445774195958128704370 9897	397702268583713330663 0233
9_{33}	0	15	30	−158435506878195970732586930756038454 3219	174738620136975814197184218325839 257
9_{34}	1	8	17	226046315631178421153	226046315631178421153
9_{35}	1	1	3	−76	19
9_{36}	2	9	20	−33778640928862345390 6162727	4395638488519
9_{37}	0	4	8	41281892	240011
9_{38}	1	5	11	−917770814359	131110116337
9_{39}	1	5	11	−290597602919	22353661763
9_{40}	0	2	4	592	37
9_{41}	0	2	4	125	5
9_{42}	1	2	5	8357	137
9_{43}	2	3	8	−76946311	5918947
9_{44}	0	5	10	−14865237107	179041
9_{45}	1	6	13	211071704505301	9043
9_{46}	0	2	4	788	197
9_{47}	0	2	4	257	257
9_{48}	1	1	3	−44	11
9_{49}	1	1	3	−23	23

Table 3: Arithmetic invariants of 10-crossing non-alternating hyperbolic knots.
NB: there is a duplication, $10_{162} = 10_{161}$, in Rolfsen's numbering.

$\mathcal{K}$	r	s	n	D	p
10_{125}	1	3	7	−3227491	10513
10_{126}	3	4	11	4354401870773	4354401870773
10_{127}	4	6	16	114610328558617087537	30554606387261287
10_{128}	2	3	8	−81723911	1151041
10_{129}	1	7	15	−3265950255260260743	20153339
10_{130}	1	5	11	−9251143919399	26142997
10_{131}	2	8	18	444056862536187349907129	336218979807407
10_{132}	1	2	5	8357	137
10_{133}	2	5	12	−32571142975223	5755635797
10_{134}	2	6	14	4399186873364681	4399186873364681
10_{135}	1	10	21	2353429531942216826098614037	865326886032362696665721
10_{136}	1	2	5	8752	547
10_{137}	0	8	16	37677571785936363184	108784045670117
10_{138}	1	3	7	−1467344	313
10_{139}	2	1	4	−688	43
10_{140}	1	3	7	−8881584	4513
10_{141}	1	3	7	−2854832	1579
10_{142}	2	2	6	452624	28289
10_{143}	3	5	13	−4149593193146416	37442243
10_{144}	0	5	10	−15937492976	996093311
10_{145}	1	2	5	9664	151
10_{146}	0	5	10	−24493056832	1352311
10_{147}	0	7	14	−112991015947556416	1765484624180569
10_{148}	3	7	17	−568459692292850719147	568459692292850719147
10_{149}	4	9	22	−199482583292876099619671811979	150693036139963
10_{150}	3	7	17	−5566349838901191567523	1668731242709
10_{151}	2	11	24	−192624291236264608283012809279991	27517755890894944040430401325713
10_{152}	3	1	5	−8647	8647
10_{153}	3	1	5	−29963	83
10_{154}	2	2	6	106229	5591
10_{155}	0	2	4	117	13
10_{156}	0	5	10	−33806471707	33806471707
10_{157}	1	1	3	−31	31
10_{158}	0	6	12	14286652240649	6552341
10_{159}	1	4	9	137768021	11549
10_{160}	1	4	9	2835227233	26987
10_{161}	2	2	6	316289	1499
10_{163}	0	5	10	−18369502259	592564589
10_{164}	0	7	14	−5077740573941683	47490582523
10_{165}	0	6	12	15124802224436	684876029
10_{166}	1	6	13	2985484001301748	746371000325437

Volume of 9_32 : 13.0998998458925268934224513554963140243326058821676687911103
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Volume of 9_33 : 13.280455636254787939973743823571002100916200587378523601643
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